PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

ICE FLOE-OFFSHORE PLATFORM INTERACTION

Bernard Ross, Consultant, Failure Analysis Associates, Palo Alto, California
Satya Hanagud, Professor, Georgia Inst. of Technology, Atlanta, Georgia
Gursharan S. Sidhu, Stanford Electronics Laboratories, Stanford California 94305
and Consultant, Failure Analysis Associates, Palo Alto, California

ABSTRACT

In this paper, the interaction of an ice sheet with an off-shore pile, such as in the support structure of a drilling platform or a docking facility, is considered. The pile is treated as a rigid member and the ice sheet is assumed to be subjected at its periphery to horizontal time-varying distributed loads. The problem is treated as a dynamic plane stress problem. The numerical solution of this model, using a large deformation computer code (CIRKICE) based on the method of artificial viscosity, yields complete stress fields in the ice sheet and the forces experienced by the pile. This information is useful both in designing the structure and in studying the problem of 'ice break-up'.

INTRODUCTION

This paper deals with the interaction of an ice sheet rigidly frozen around a pile that may be part of the support structure of a docking facility or an off-shore platform. The ice sheet is subjected to various external load factors such as wind stresses, water stresses, and other forces because of impingement by either a stationary or moving sea ice pack.

If one supposes that the ice is initially quiescent it appears that there might be three distinct phases of interest. The occurrence of these successive phases would correspond to the ice sheet being forced against the pile by ocean currents, or distant stresses in the ice sheet, of progressively increasing strength.

In the first phase, the stresses in the surrounding ice sheet increase, until the stress at some point in the ice body becomes sufficiently great for initial failure to occur. Strains will probably be small during this prefailure phase of the loading, so that the analysis need not be nonlinear on this account. However, significant nonlinearity can be present in the stress-strain relations. In the second phase the process of ice break-up progresses to completion, accompanied by the generation of new boundaries and consequent redistribution of stress. This continues until in the third phase the ice sheet fractures into smaller pieces which "flow" past the pile. This paper is concerned with the first of these three phases.

It would be appropriate to solve this first phase problem for linear, elastic-plastic material behavior with strain hardening included to clarify the sea ice sheet response after yield. This formulation has proved successful in previous analytical studies [2,4] where the penetration and perforation of an Arctic sea ice cover by impacting projectiles was studied and good correspondence of theoretical results with experimental data noted. In this earlier work, viscoelastic material behavior was not considered; however, it would present little additional difficulty to acknowledge sea ice response of this type modeled by a series Maxwell-Voigt representation for viscoelastic sea ice behavior.

The pile is treated as a rigid circular member of outer radius, a, with an ice sheet of arbitrary shape frozen rigidly to its surface (see Fig. 1). The ice sheet is assumed to be large in extent, homogeneous, and isotropic; its thickness is small compared to the pile diameter and can vary as a function of (x, y) and time t.

The objective of this study is to describe quantitatively the loading configuration, stresses, and displacements in the ice sheet and on the pile, and the likelihood of fracture and break-up of the ice. The problem is modeled as a plane stress, time dependent one, with elastic-plastic ice behavior. The model has been theoretically analyzed using an adapted version of an existing two-dimensional, large deformation, elastic-plastic computer code (CANDIA CODE) [2, 3, 4]. The determination of plane stress states is done subject to appropriate constitutive and boundary conditions introduced into a code formulation which includes provisions for incorporating nonlinear material behavior. Boundary conditions are cast in a Cartesian coordinate system and acknowledge an outer boundary surface not in coincidence with the finite difference modes. Although a circular polar coordinate system would simplify boundary condition treatment, it is believed that this approach introduces calculation inaccuracies due to the fact that the requisite finite difference mesh network size must increase as a function of radius away from the piling. Although use of Cartesian coordinates at the boundary can introduce errors, these errors will be of the same order as those contributed by the finite difference representatives of the partial differential equations.

As byproduct, important insights into other aspects of ice behavior can be obtained. Such a study can lead to better analytical understanding of the impact and/or bearing conditions that lead to the formation of pressure ridges in interacting ice bodies. In this context, determination of above ice and below ice heights and slope angles and/or base widths of theoretical pressure ridge morphologies could be sought.

BASIC EQUATIONS

Basic equations for the present large deformation dynamic response problem under conditions of plane stress can be written in the following way.

a. Equations of motion

$$\frac{\partial}{\partial x}(h\sigma_x) + \frac{\partial}{\partial y}(h\tau_{xy}) = \rho\dot{u} \qquad (1)$$

$$\frac{\partial}{\partial x}(h\tau_{xy}) + \frac{\partial}{\partial y}(h\sigma_y) = \rho\dot{v} \qquad (2)$$

b. Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{h}\dot{h} = -\frac{1}{\rho}\dot{\rho} \qquad (3)$$

c. Stress-Strain Relationship

$$\dot{\epsilon}_x = \frac{1}{E}(\dot{\sigma}_x - \nu\dot{\sigma}_y) \qquad (4)$$

$$\dot{\epsilon}_y = \frac{1}{E}(\dot{\sigma}_y - \nu\dot{\sigma}_x) \qquad (5)$$

$$\dot{\gamma}_{xy} = \frac{1}{G}\dot{\tau}_{xy} \qquad (6)$$

$$\dot{\epsilon}_z = -\frac{\nu}{E}(\dot{\sigma}_y + \dot{\sigma}_x) \qquad (7)$$

d. Kinematics

$$\dot{\epsilon}_x = \frac{\partial u}{\partial x} \qquad (8)$$

$$\dot{\epsilon}_y = \frac{\partial v}{\partial y} \qquad (9)$$

$$\dot{\epsilon}_z = \frac{1}{h}\dot{h} \qquad (10)$$

$$\dot{\gamma}_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad (11)$$

In these equations, $x = x(\alpha, \beta, t)$ and $y = y(\alpha, \beta, t)$ denote the x and y coordinates of the ice sheet element whose initial position at t = 0 is given by $x = \alpha$ and $y = \beta$. Dots indicate differentiation along the particle path while u and v represent the particle velocities. Stresses and strains in the xy-plane are represented by $\sigma_x, \sigma_y, \tau_{xy}$ and $\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}$ respectively. Density, which can vary in both space and time is given by $\rho = \rho(x, y, t)$

BOUNDARY CONDITIONS

It is assumed that the ice sheet is fully fixed to the rigid inner pile. Thus, u and v are equal to zero at r = a. At the outer boundary, arbitrary forces can be prescribed in the plane of x and y. Even though the equations can be used to consider an ice sheet of arbitrary shape, the initial stage of the programming has treated an ice sheet that is initially circular (r = b) at time, t = 0. It must be noted that deviations from this original configuration take place as cumulative deformation occurs. If n and t denote the normal and tangent directions to the outer boundary, the stresses $\sigma_n(x,y,t)$ and $\tau_n(x,y,t)$ can be prescribed. The present version of the program considers only $\sigma_n(x,y,t)$ at the outer boundary, while $\tau_n(x,y,t) = 0$

METHOD OF INTEGRATION

Equations governing the problem have been represented in finite difference form. These difference equations have been solved numerically by the method of artificial viscosity.[1] The finite difference treatment and integration procedure have been described thoroughly by the authors in their earlier papers.[2,3,4]

COMPUTER PROGRAM

Numerical representation of a circular boundary in a Cartesian mesh work is illustrated in Figure 2. Figure 2 also shows the block diagram of the plane stress dynamic code which has been named CIRKICE. This block diagram presents the flow chart of the program and the calls to the various subroutines. Also provided is a block diagram of the CANDIA CODE [2] for purposes of comparison.

Finally, the following properties of ice were used in running test problems and computer check outs.

Properties of Ice

Density	ρ	=	0.918 gm/cm^3
Poisson's Ratio	ν	=	0.29
Young's Modulus	E	=	273,000 psi
Rigidity Modulus	G	=	105,800 psi
Bulk Modulus	K	=	217,000 psi
Speed of Sound	c	=	0.0645 in/μ sec
Yield Strength	Y_t	=	140 psi

RESULTS

Numerical results for a sea ice-structure interaction problem of practical importance are given in Figure 3. Here, a long, vertical, rigid, circular cylindrical pile, 4 ft. in diameter, is surrounded by a circular sea ice sheet, 12 ft. in diameter. The ice sheet is subjected to a symmetrical uniaxial displacement field directed along the negative x-axis. This pattern varies as the cosine of the angular coordinate (e.g. $U = U_o$, o when Θ = o, $\pm\pi/2$ respectively) and is applied to one half the outer periphery of the ice sheet, the other half remaining as a stress free boundary. Thus, the configuration under study corresponds to sea ice impingement on a pile due to a uniform one dimensional ocean current.

Values of the plane stress quantities: $\sigma_x, \sigma_y, \tau_{xy}$ as a function of the circumferential coordinate, Θ, are provided. It is noted that corresponding values of radial, σ_r, and circumferential, σ_Θ, normal stresses as well as the associated shear stress, $\tau_{r\Theta}$, can be obtained readily from the following conversion relationships

$$\sigma_r = \sigma_x \cos^2\Theta + \sigma_y \sin^2\Theta + 2\tau_{xy} \sin\Theta \cos\Theta$$

$$\sigma_\Theta = \sigma_x \sin^2\Theta + \sigma_y \cos^2\Theta - 2\tau_{xy} \sin\Theta \cos\Theta$$

$$\tau_{r\Theta} = (\sigma_y - \sigma_x) \sin\Theta \cos\Theta + \tau_{xy}(\cos^2\Theta - \sin^2\Theta)$$

For the present problem, it is determined that $\sigma_r = 0$ when $\Theta = 119^o$. Therefore, if ice-pile adhesion stresses are omitted from consideration, breakaway of the ice sheet from the pile will commence at these two angular locations and the pile will be stress free over its rear surface through an included angle of 122^o. Under this condition, strong asymmetrical load distribution in the pile cross section plane will produce greater structural bending stresses than in the completely frozen-in configuration.

Fracture and breakup of the sea ice sheet can also be predicted by using an appropriate failure criterion for the combined stress state (Tresca, Von Mises, etc.) and introducing values for the component stresses as given in Figure 3. For example, crushing failure due to compressive bearing stresses is predicted along the x-axis where $\sigma_r \approx \sigma_x$ when

$$U_o^* = \frac{\sigma_c}{73} \times 0.01$$

where U_o^* is the amplitude of displacement for failure at a critical crushing stress for sea ice, σ_c.

REFERENCES

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FIGURES

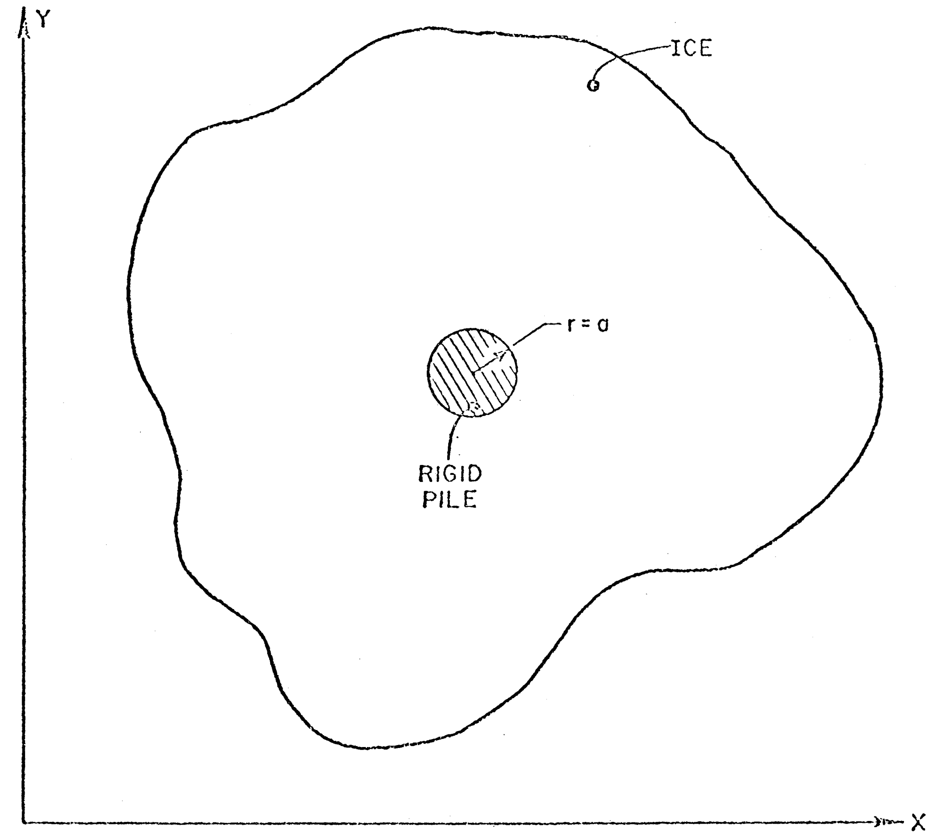


Fig. 1

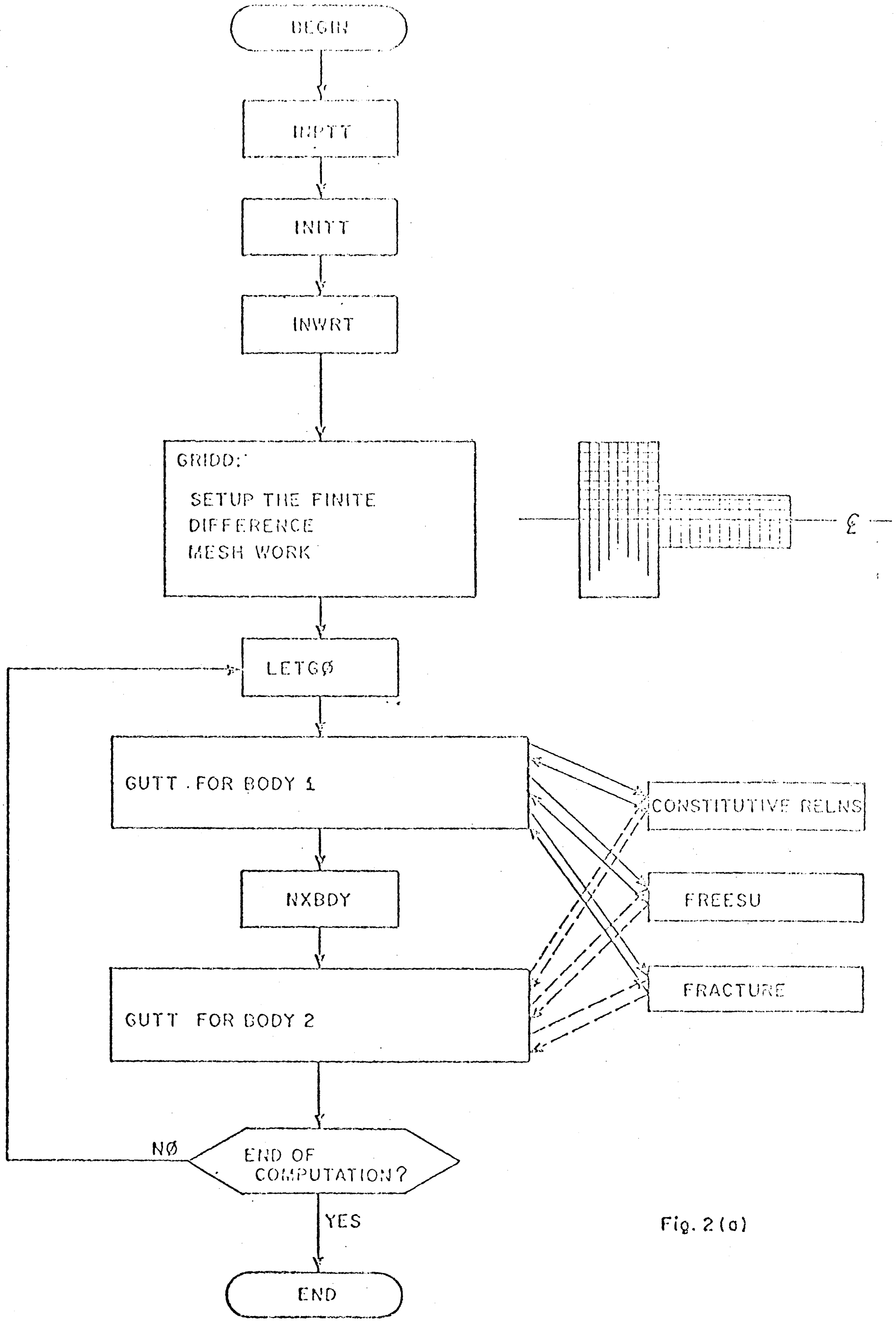
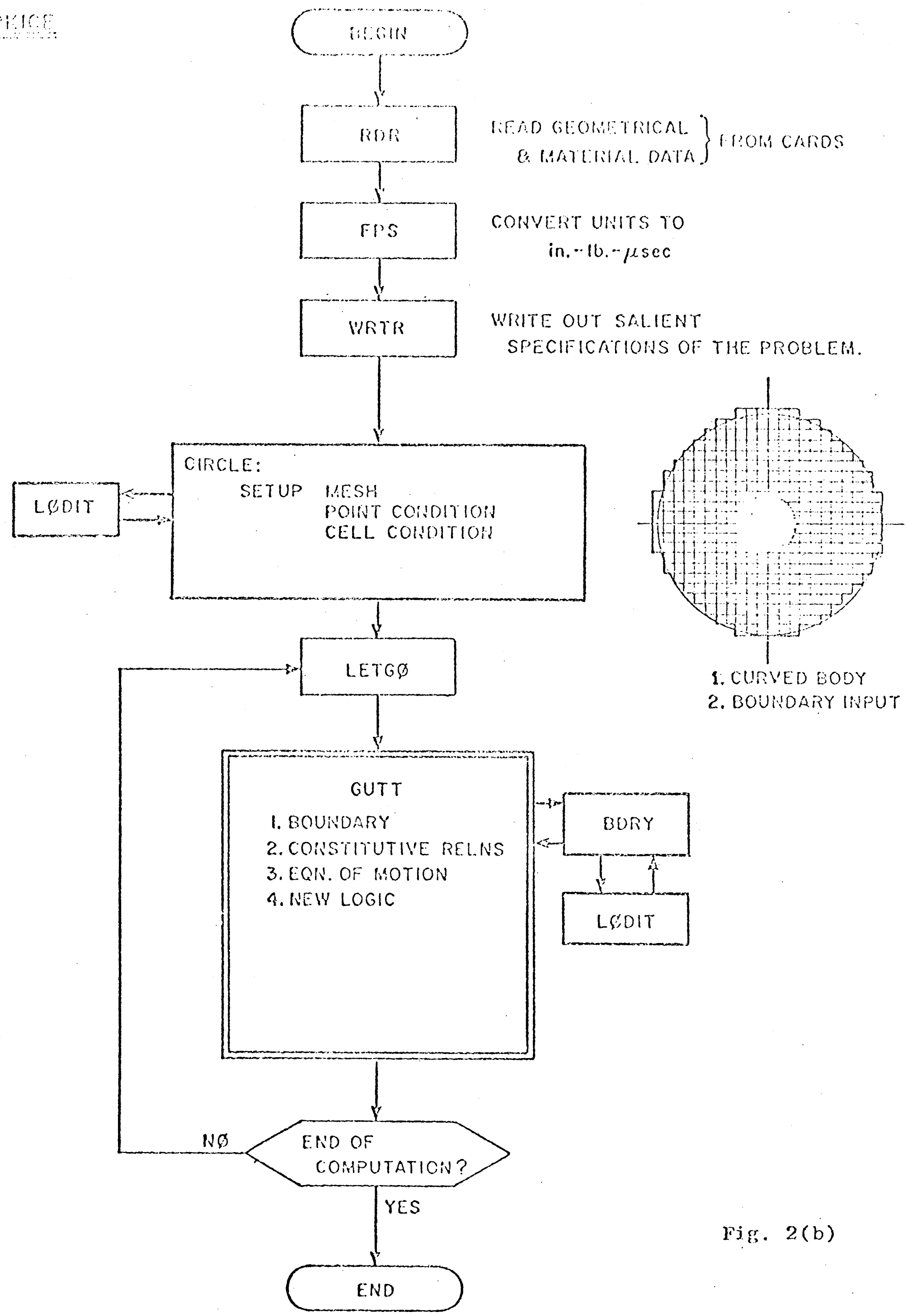

BEGIN
INPTT
INITT
INWRT
GRIDD:
SETUP THE FINITE
DIFFERENCE
MESH WORK
℄
LETGØ
GUTT . FOR BODY 1
CONSTITUTIVE RELNS
NXBDY
FREESU
FRACTURE
GUTT FOR BODY 2
NØ
END OF
COMPUTATION?
YES
END

Fig. 2 (a)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FROM CARDS
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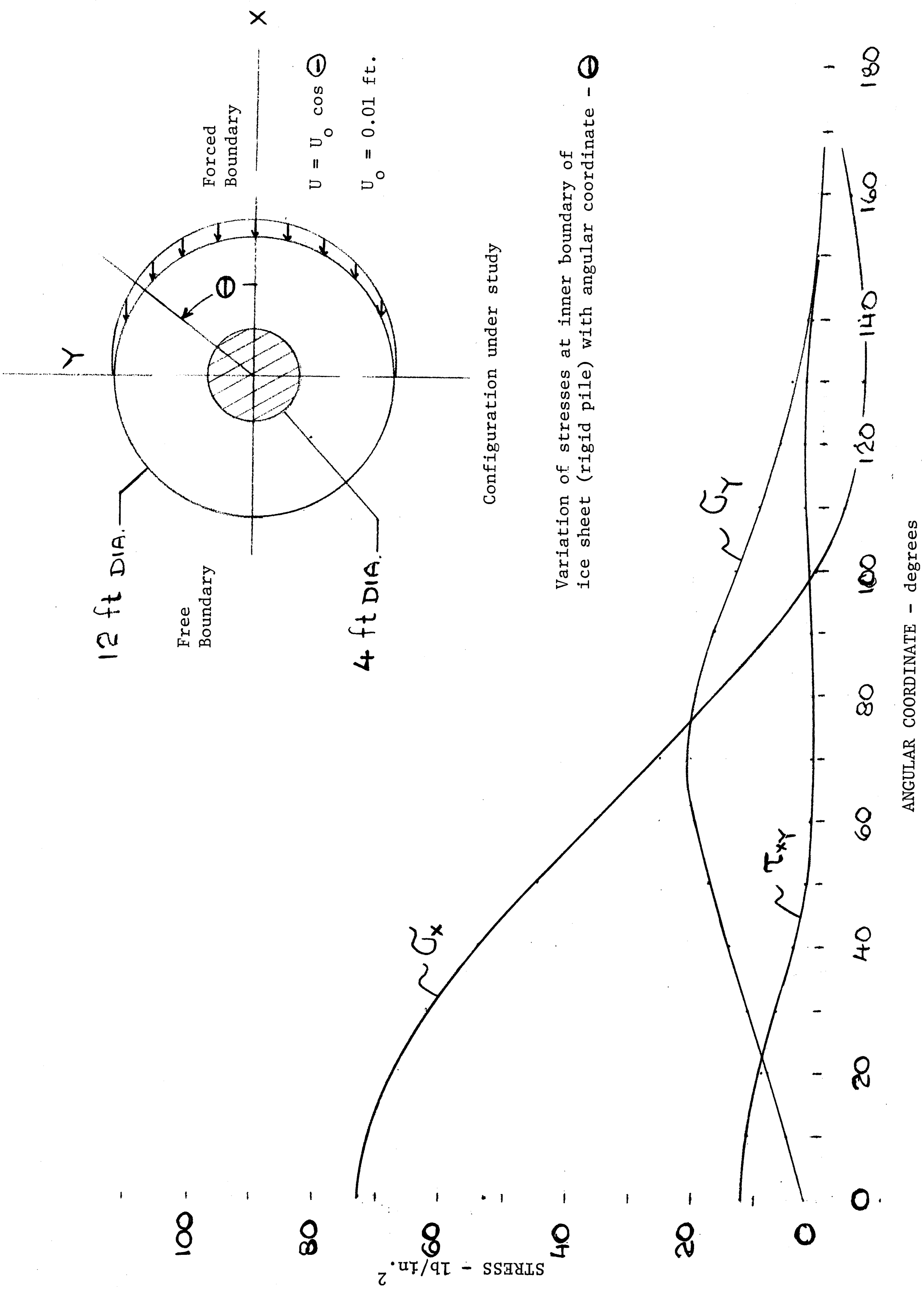
CONVERT UNITS TO in.-lb.-μsec
WRTR
WRITE OUT SALIENT SPECIFICATIONS OF THE PROBLEM.
CIRCLE:
SETUP MESH
POINT CONDITION
CELL CONDITION
LØDIT
LETGØ
1. CURVED BODY
2. BOUNDARY INPUT
GUTT
1. BOUNDARY
2. CONSTITUTIVE RELNS
3. EQN. OF MOTION
4. NEW LOGIC
BDRY
LØDIT
END OF COMPUTATION?
NØ
YES
END

Fig. 2(b)

FIGURE 3



Configuration under study

Variation of stresses at inner boundary of ice sheet (rigid pile) with angular coordinate - Θ