PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

THE INTERPRETATION OF SMALL SCALE STRENGTH DATA FOR ICE

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ABSTRACT

It is recognized that certain effects observed in the testing of small samples of ice are not well understood. It is proposed that these effects could be better understood by examining more carefully the stress and deformation behavior experienced by a small sample. The relatively large grain sizes and the plasticity of the individual grains are identified as critical factors in this examination. Two types of models are proposed, each of which models a small sample as an assembly of unique single crystals. One model represents each grain by a spring and dashpot, while the other model represents the granular structure using the finite element technique. It is shown that both models are able to reproduce an experimentally observed strength-strain rate effect.

INTRODUCTION

Most engineers who are concerned with navigation, transportation, port facilities and offshore structures in the waters of cold climates have, at one time or another, encountered problems related to ice forces and ice strength. Although such problems have presented themselves frequently, the amount of usable information available on this subject is comparatively small. This is so because of the complex behavior of ice as the material component of natural ice sheets. For example, it is known that the strength of a given ice sheet depends on its thickness, on the area and shape of the structure upon which it bears, on the surface temperature, on the water salinity (where applicable), and on its rate of motion. In addition, an ice sheet will fail in one of several modes depending upon the above variables. It can crush into small pieces, or crack in plane stress, bending, or buckling.

Given this large list of strength variables, and given the cost of conducting full scale tests, the idea of developing strength relationships for ice sheets from full scale strength tests has not been pursued to any great extent. Rather, ice strength investigations have been on small scale samples, with the hope that these small scale results will eventually apply to the full scale ice sheet via experimental or analytical modeling techniques. At present, this hope has yet to be realized. The amounts of small scale strength data which are currently available are sufficiently inconclusive amongst themselves that they have not been applied to full scale ice sheets. Unless the results of small scale strength tests are better understood, their application to engineering problems cannot be realized.

The purpose of the research described herein is to take a closer look at the mechanical behavior of ice in a small sample strength test. Particular attention is given to the effect of granular structure and plasticity on the strength results. It is shown that when these two factors are incorporated into the ice behavior via two simple models, some of the observed strength results can be explained.

A REVIEW OF SMALL SAMPLE TESTING

Ice Structure

Before reviewing the results of small sample tests, it is necessary to describe briefly the structural properties of ice.* Natural ice exists as a

* A complete review is presented by Weeks and Assur[1].

coarse polycrystalline agregate, with typical grain diameter ranging between 0.1 and 2.0 cm. (2 to 3 orders of magnitude larger than that for most metals). The individual crystals (grains) consist of planes with a hexagonal arrangement of oxygen atoms, stacked above one another. The plane of hexagonal symmetry is referred to as the "basal plane", and the axis perpendicular to the basal plane is referred to as the "c axis". Due to conditions of growth in an ice sheet, the crystals themselves tend to form in certain regular geometric patterns. The most frequently encountered regularity is that of "columnar-grained" ice, depicted in Fig. 1. This type of structure is characterized by grains of columnar shape, with the column axis perpendicular to the plane of the ice sheet. The c axes of the grains tend to lie in the plane of the ice sheet.

Sample Characteristics

Figure 2 shows some of the most typical sample configurations. The *in situ* cantilever, although not a small scale test, is included here because of the quantity of data available for such tests. Figure 3 shows the most typical orientation of grains with respect to samples for tests on columnar-grained ice. The ensuing discussion, unless otherwise stated, will refer to these configurations.

Summary of Strength Results

A comprehensive review of small sample strength test results is presented by Weeks and Assur[1]. For the purpose at hand, those results shall be presented which best illustrate some of the problems which have arisen in the understanding of these test results.

Uniaxial compression - Aside from the fact that ice is typically about four times stronger in uniaxial compression than in uniaxial tension, the most interesting aspect of these tests is the occurrence of internal cracking at loads well below the ultimate failure load. Such cracking is time dependent and load dependent.

Tension - Tests for tensile strength include the uniaxial tensile test, small beam bending test, the ring tensile test, and the *in situ* cantilever. Although each test theoretically measures the same quantity, results for the different types of tests have never agreed, and usually follow the following inequality:

$$\sigma_{rt} > \sigma_{fl} > \sigma_t > \sigma_{isc} \qquad (1)$$

where σ_{rt} is the ring tensile strength, σ_{fl} is the small beam flexual strength, σ_t is the uniaxial tensile strength, and σ_{isc} is the flexual strength of an *in situ* cantilever beam. It should be noted that σ_t is the only directly measured tensile strength. The other quantities are computed from the load via the elastic beam formula in the case of σ_{fl} and σ_{isc}, and via the elastic stress solution for a disc with a hole in the case of σ_{rt}. Since failure in ice is usually governed by tension, and since the uniaxial tensile test is difficult to conduct and impractical as a field test, the above disagreement in results poses a serious obstacle to the application of these tensile results to the study of ice sheet strength.

Load rate - One important variable whose effect on strength is least understood is the load rate (or strain rate). Although this effect was not studied until recently, field observations of ice failing against a structure show that the force applied to the structure depends heavily on the rate of motion of the floe.[(2)] The study of the load rate or strain rate effect on small samples has produced some interesting results. The results of rate studies for uniaxial tensile and compression tests on both lake and sea ice are shown in Figs. 4, 5, 6 and 7. The results of different investigators have been plotted on the same graph for the sake of broadening the observable rate regime.

It is suggested from these figures that ice in a uniaxial test falls into one of two rate regimes - one where strength increases with increasing rate and one where strength decreases with increasing rate. With the exception of the results of Jellinek (which are for fine grained equigranular ice), no one test series spans both rate regimes. Hence the above suggestion remains speculative, and there is no information concerning the location of the transition point (if one really exists).

Paige and Kennedy have studied the effect of the crosshead speed of the testing machine on the computed ring tensile strength. Their results are shown in Fig. 8. As can be seen, this strength-rate curve is quite different from those found in uniaxial tests. In addition, the strength values approached at the high crosshead speeds are much lower than those obtained from typical ring tensile tests, which are conducted at much lower speeds.

Size effects - It is relevant in this discussion to mention the observed relationships between grain size, sample size and strength. An examination of the inequality (1) gives a first indication that the small sample tests yield higher values of strength than the full size in situ cantilever. Of greater interest here is the size-strength effect for small samples of similar type. In uniaxial compression on fresh ice, Butkovich[(9)] found strength to decrease with both increased cross sectional area and decreased grain size. Jellinek[(7)] has found strength to decrease with increased sample volume for fine grained snow ice in tension. In sea ice, grain size is found to increase with increasing depth in the ice sheet. Hence measurement of strength vs. depth with other variables fixed has been used to measure strength vs. grain size. With this interpretation, Dykins[5] found no variation of strength with grain size in a uniaxial tensile test, while Langleben[10] found strength to increase with increased grain size for ring tensile tests.

Discussion of Small Sample Test Results

The three significant areas of doubt surrounding the results of small sample tests deal with strength vs. type of test, strength vs. load or strain rate, and strength vs. grain size and sample size. These doubts can be resolved by taking a closer look at the stress and deformation behavior within the small sample, and by reevaluating some of the assumptions which have been made in order to produce the reported "strength" values.

The first assumption to be questioned is that of elastic behavior. In both flexural and ring tensile tests, strength has been computed from a formula derived from elastic theory. It is well known (e.g., from the flow of glaciers) that ice exhibits some sort of time dependent plastic deformation. Up until recently, however, it has been assumed that at loading rates above 0.5 $kg/cm^2/sec$. such deformation becomes insignificant. This criterion was derived by Butkovitch[11] from the results of Jellinek (Fig. 7) which show that tensile strength is independent of rate in this regime. For tests such as the ring tensile test, the stress rate can only be computed using the assumption of elasticity, and hence considering the possibility of plastic behavior implies that the stress rate is indeterminate. Secondly, the results of Jellinek are for fine grained ice not at all typical of the large grained ice found in natural ice sheets. This leads to a second questionable assumption. The dimensions of most small samples have been such that the individual grains represent relatively large units in the small sample. Thus it is questionable to analyze a small sample as a homogeneous material. The fact that strength has frequently been found to decrease with increasing sample size and decreasing grain size suggests that the number of grains per sample is an important parameter in determining strength.

The results of Paige and Kennedy (Fig. 8) shed some light on the above discussion. Recognizing that the stress rate could not be directly measured, they conducted a series of ring tensile tests at various crosshead speeds of the testing machine. Their results suggest that plasticity relieves the stress concentration at the lower speeds, as shown in Fig. 9, thus resulting in an overcomputation of the ring tensile strength. The ring tensile strength for the high speeds approaches a value more typical of that found in uniaxial tension. This argument offers an explanation for the consistently high ring tensile strength values which have been reported (see Eq. 1).

The above argument also yields some explanation for the role of the individual grains in the behavior of an ice sample. The elastic stress concentrations which would be expected to occur at points A and B, Fig. 10, are relieved by the plastic behavior of the individual grains at these locations. Hence the plastic deformation which characterizes the deformation and failure of the sample is that of the individual crystals rather than that of the polycrystal as a whole. This is an important distinction, since, as will be discussed shortly, the plastic behavior of ice single crystals is much different from the behavior of polycrystalline ice.

Although the ring tensile test represents a more obvious manifestation of the single crystal plasticity, one should expect that all types of small samples are governed by the interaction of a relatively small number of elasto-plastic grains. The view presented here is that the phenomena of strength vs. type of test, strength vs. load rate, and strength vs. sample size and grain size can be understood in view of the above behavior. The remainder of this paper will be devoted to the development of models which can analyze an ice sample as an assembly of elasto-plastic grains.

PLASTIC BEHAVIOR OF SINGLE CRYSTALS OF ICE

Before developing models based on single crystal behavior, a brief review of the current knowledge of single crystal mechanical properties will be presented.

Geometry of Deformation

It has been found that the predominant form of plastic deformation in single crystals occurs when there is a resolved shear stress on the basal plane.[12][13] Deformations on other slip systems are two orders of magnitude less than basal slip for the same magnitude of loading.[14] Hence the following results will concern basal slip alone.

It has been demonstrated that there is no preferred slip direction in the basal plane,[15] i.e., basal slip occurs in the direction of maximum resolved shear stress. Ice crystals deform like a stack of plates sliding over one another. Figure 11 illustrates some of the deformation characteristics which have been observed.

Creep, Stress-Strain, and Relaxation Characteristics

Figure 12a shows typical creep, stress-strain, and relaxation curves found for single crystals of ice. The unusual behavior in creep and stress strain is worthy of note. Unlike polycrystalline ice, which exhibits the usual three-stage creep, single crystals exhibit only accelerating creep. Stress-strain curves at constant strain rate exhibit a yield point followed by a yield drop, or strain softening. The value and location of the yield point varies with temperature and strain rate.

Mathematical Description of Single Crystal Flow

Relationships between stress, strain, and time have been developed both empirically through creep, stress-strain, and relaxation tests, and theoretically through dislocation theory. The aim here is to obtain a single relationship governing the plastic flow of single crystals of ice which can reproduce observed creep, stress-strain, and relaxation results.

The following relationship, presented by Readey and Kingery[18], serves as a starting point:

$$\dot{\gamma}_p = k\gamma_p^m \tau^n \qquad (2)$$

Here, γ_p is the plastic shear strain and τ is the shear stress, both resolved onto the basal plane.* Considerations of dislocation theory[19] suggest that the total plastic strain, γ_p, is the sum of an initial plastic strain, γ_o, and a measured or applied plastic strain, γ_{pm}, i.e.:

$$\gamma_p = \gamma_{pm} + \gamma_o \qquad (3)$$

If τ/G represents the elastic strain (G is the shear modulus) and γ_T represents the total strain developed, then:

$$\gamma_{pm} = \gamma_T - \tau/G \qquad (4)$$

Substituting (3) and (4) into (2), the following relationship between stress, strain, and time for basal glide is obtained:

$$(\dot{\gamma}_T - \dot{\tau}/G) = k(\gamma_T + \gamma_o - \tau/G)^m \tau^n \qquad (5)$$

* k, referred to here as the flow modulus, is a function of temperature, (and in the case of sea ice, salinity or brine volume) and m and n are exponents, considered for the moment to be material constants.

where $(\dot{\ })$ represents $d(\ )/dt$

The above relationship suitably approximates observed behavior in creep, relaxation, and stress strain at constant strain rate. Experimental results of Readey and Kingery[18], and of Higashi et al.[20] show that k depends on absolute temperature T as follows:

$$k = k' \exp(-Q/RT) \qquad (6)$$

where Q is approximately 15 kcal/mole, R is the universal gas constant, and k' is a material constant. Comparison of (5) with experimental results shows that $0 < m \leq 1$ and $1.5 < n < 4.0$.

Equation 5 will henceforth be used to describe single crystal plastic deformation. Solution stress-time curves at constant strain rate are shown in Fig. 12b. It can be seen that the yield point increases with increasing strain rate and decreasing temperature (decreasing k). Since γ_o is difficult to quantify, it will be treated here as a material constant.

BEHAVIOR OF SINGLE CRYSTALS IN A POLYCRYSTAL

In the discussion of single crystal behavior of the last section, the experimental samples from which results were obtained were entire single crystals. It is desired to use these results to predict the behavior of single crystals when they are acting as components of a polycrystalline ice sample. Gold has conducted a series of creep compression tests on columnar grained ice,[21][22][23] and his observations regarding the behavior of the grains can be summarized as follows.

1) Basal glide cannot accommodate the arbitrary change of shape to which each grain, under the influence of its neighbors, must conform.
2) As a result, localized elastic stress concentrations develop.
3) These stresses are relieved by internal cracking (which occurs well below the failure load).
4) These cracks occur primarily within the grains, most frequently parallel to the basal plane; less frequently, perpendicular to the basal plane.

The above observations provide a starting point for the development of models for the behavior of polycrystalline ice.

Spring Dashpot Model

A simple approach to study the dependency of small sample failure on grain behavior is to treat a uniaxial tensile sample with the grains at the critical section represented as a system of springs and dashpots (Fig. 13). If F_i represents the total force acting on grain i, and δ represents the strain at the cross section (assumed constant for the cross section), then the relation for stress and strain in a single crystal (Eq. 5 at constant strain rate v with m = 1) leads to the following force-displacement relation for the equivalent spring-dashpot system.[24]

$$\frac{dF_i}{d\delta} = G_i - \frac{G_i k_i}{v} \left(\delta + \delta_i - \frac{F_i}{G_i}\right) F_i^n \qquad (7)$$

where $G_i = 2A_iG$

A_i = cross sectional area of grain i

$k_i = \left(\frac{\sin 2\Theta_i}{2A_i}\right)^n \bar{k}$

$\delta_i = \gamma_o / \sin 2\Theta_i$

Θ_i = angle between c axis and specimen axis

To illustrate qualitatively the type of behavior predicted by such a model, the force capacity vs. strain rate is examined for two adjacent grains (Fig. 14). The resulting force displacement curves depicting the relative parameters of the two grains show that grain 1, which is more favorably oriented for plastic flow, yields at a lower strain than that for grain 2. Failure of the two-grain system is expected to occur with the failure of grain 2, at $F_2 = F_2^*$ and $\delta = \delta_2^*$. This is because grain 2 experiences the maximum tensile stress across the basal plane. Since the yield point of grain 1 increases with increasing strain rate v, and since grain 2 is still in its elastic range at failure, the load capacity of the two-grain system increases with increasing strain rate. Should this observation apply to the entire cross-section, then it would provide an explanation for some of the observed strength-strain rate (load rate) experimental results.

This model was pursued further by using Eq. 7 to develop force-deformation relationships for a cross section with N grains. To include internal cracking as part of the deformation process, individual grains were assumed to fail when either the tensile stress on the basal plane reached a critical value σ_{bc} or when the tensile stress on the plane perpendicular to the basal plane reached a critical value σ_{pc}. From the observations of Gold, σ_{pc} was taken arbitrarily to be 1.5 σ_{bc}. This analysis was carried out on the IBM 1130 computer. Figure 15 shows typical force-deformation curves for a 6-grain cross section at different strain rates. With the maximum force representing the strength of the sample, such simulations were used to develop strength vs. strain rate curves. Examples of the curves obtained are shown in Fig. 16. Each point on these curves represents the average of ten randomly simulated samples. These curves show that the spring dashpot model can reproduce observed uniaxial tensile strength vs. strain rate results for the lower load rate regimes.

Finite Element Model

Since the spring dashpot model does not represent the restraints imposed on each grain by its neighbors, a finite element model was developed to represent more accurately the behavior of grains in a polycrystalline sample. The finite element method, by definition, divides a continuum into discrete elements, and hence it is a natural approach for the analysis of a polycrystal as an assembly of single crystals.

Basically, the method used here employs plane strain elements, as applicable to problems with columnar grains. Each grain was assembled from square and 45^{o} triangular elements, and the c axis orientation was chosen randomly in the plane of the element by the computer. The basic finite element method for an elastic continuum was extended to apply to the elasto-plastic behavior of ice crystals. A more detailed description of the procedure is presented by Maser.(25)

Using this method, the model shown in Fig. 17 was analyzed for force deformation relationships at different strain rates. Resulting force-deformation curves are shown in Fig. 18. The non-uniform curvatures which are immediately apparent are manifestations of both the instability of grains which have reached impending yield but are restricted by neighboring grains, and the integration interval, which here is 0.01 inches. As expected, the force-deflection curves approach a straight line with increasing strain rate for a given sample. From the stress values obtained with the computer program, it was determined that the maximum value of σ_{bc} occurred at point A in Fig. 16. Due to the orientation of the grain at point A, σ_{bc} was relatively insensitive to v, and hence its value at a given deflection, δ, was the same at the different strain rates. An arbitrary value of σ_{bc} was chosen coinciding with $\delta = 0.07$, and from this value and the curves of Fig. 18 a strength vs. strain rate curve was obtained (Fig. 19). This curve has the same form as those found with the spring-dashpot model (Fig. 16) and hence reproduces the low rate regime of observed experimental data on uniaxial tensile tests.

DISCUSSION OF RESULTS

Through the use of a spring-dashpot model and a finite element model, an important strength-strain rate effect has been analytically reproduced. Both models are considerable idealizations of reality. The spring-dashpot model represents a highly simplified version of the grain interaction and the resulting stress state. The finite element model, although presenting a detailed stress representation for a more realistic geometric arrangement, treated failure to be initiated by the failure of a critically stressed grain. Nevertheless, their coincidence encourages the idea that the rate phenomenon observed is a function of the plastic properties of the grains, and that the phenomena of strength vs. type of test and strength vs. sample and grain size can be understood in the same manner with these models. Work is currently in progress to achieve this end.

It should be noted that the experimentally observed decrease of strength with increasing strain rate at high strain rates did not manifest itself in the models presented here. One must conclude that this phenomenon is due to a rate mechanism other than plasticity -- most likely that associated with brittle fracture propagation in a polycrystal.

CONCLUSIONS

It has been shown here that some of the results associated with the strength of small samples of ice can be understood in terms of the plastic properties of ice single crystals. A particular strength-strain rate effect for uniaxial tensile tests has been analytically simulated in these terms using both a spring dashpot model and a finite element model for the behavior of the individual grains in a polycrystalline sample. This suggests that other small sample test results can be explained in a similar fashion. Such an understanding of the small sample data which is currently available would be very useful in the modeling of ice properties in a variety of ice engineering problems.

ACKNOWLEDGMENTS

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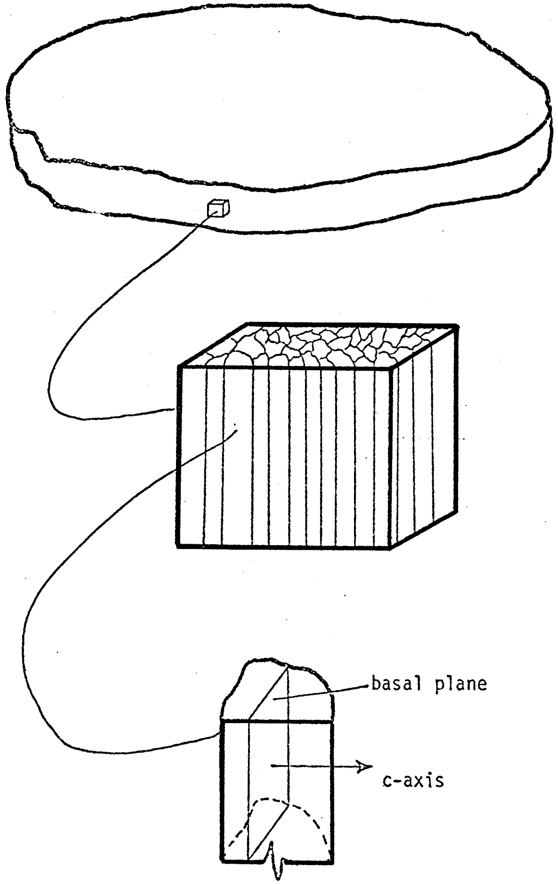


Figure 1 Grain Structure in an Ice Sheet

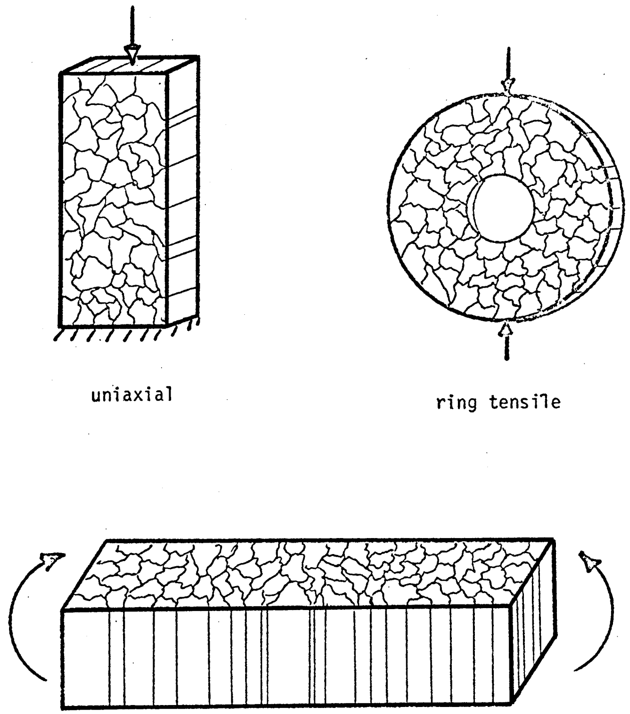


Figure 3 Orientation of Grains with respect to Specimens

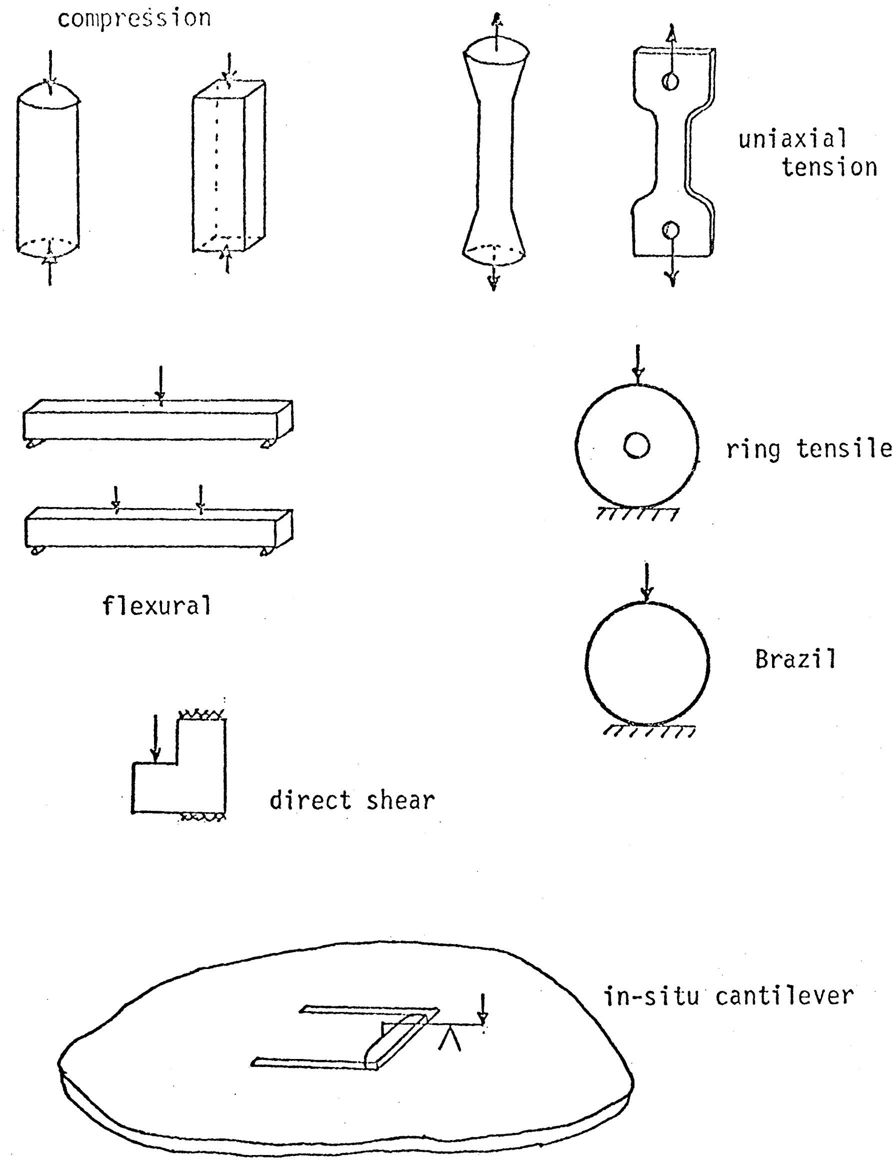


Figure 2 Geometry of Tests

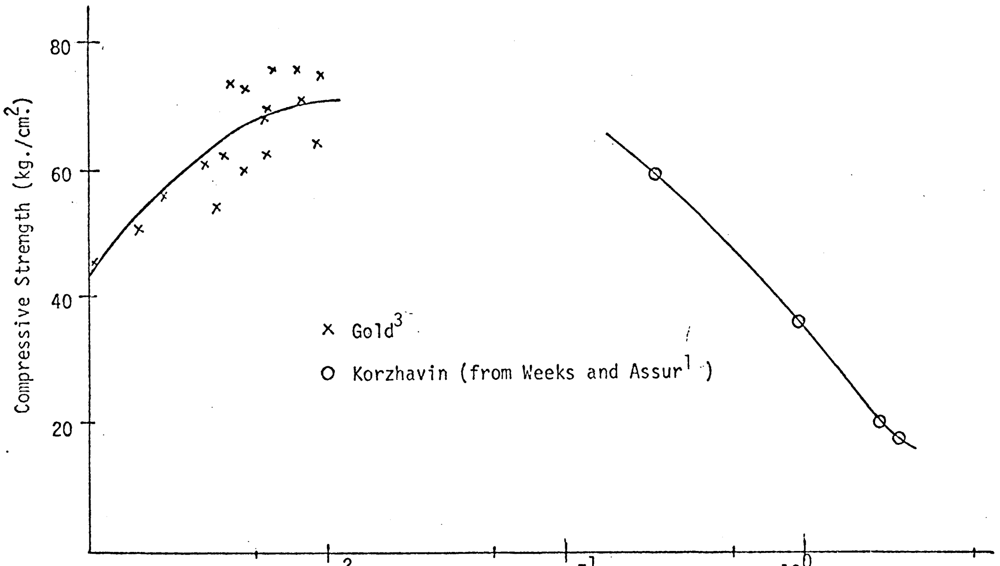


Figure 4 Compressive Strength vs. Strain Rate (fresh ice between -9°and -11°C)

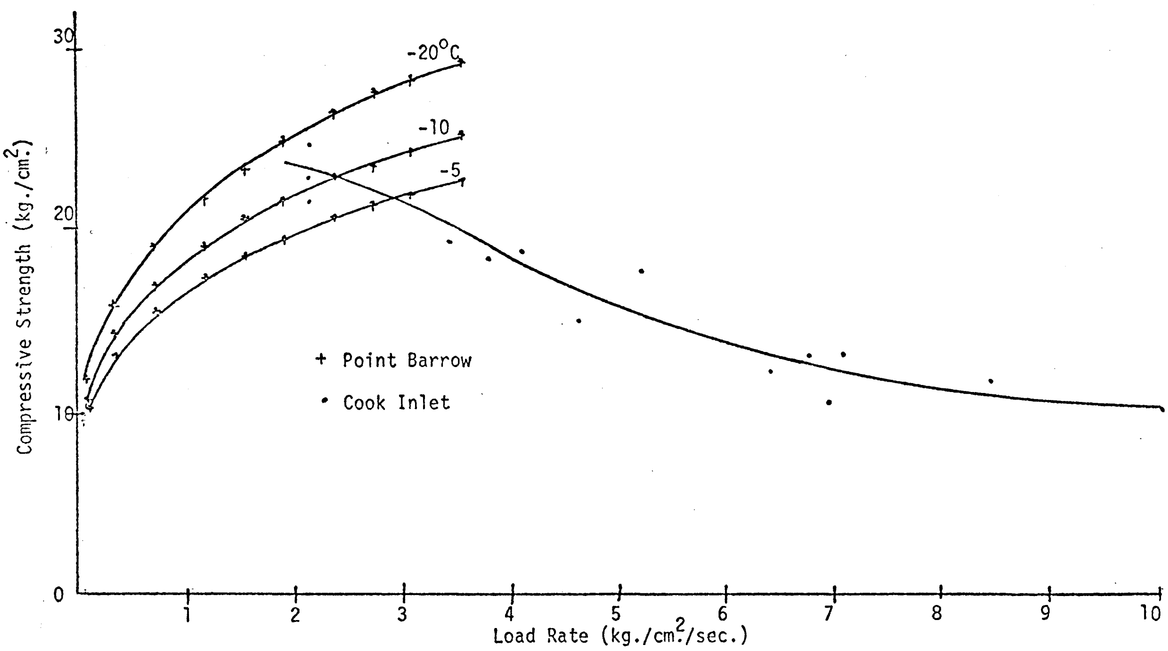


Figure 5 Compressive Strength vs. Load Rate (Peyton[4])

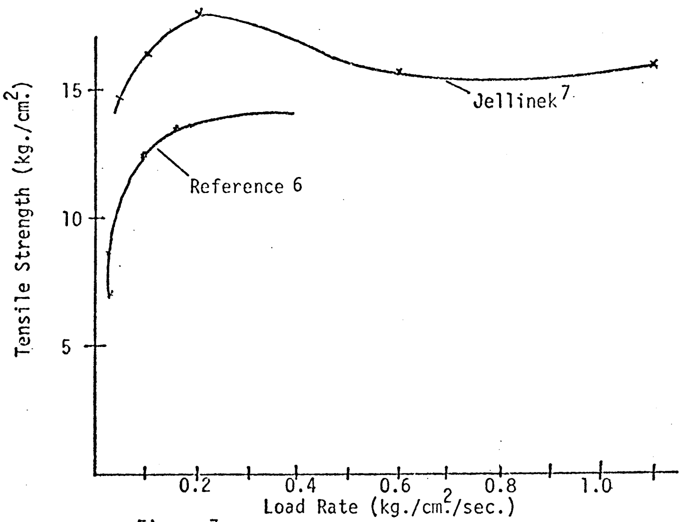


Figure 7
Tensile Strength vs. Load Rate-Fresh Ice

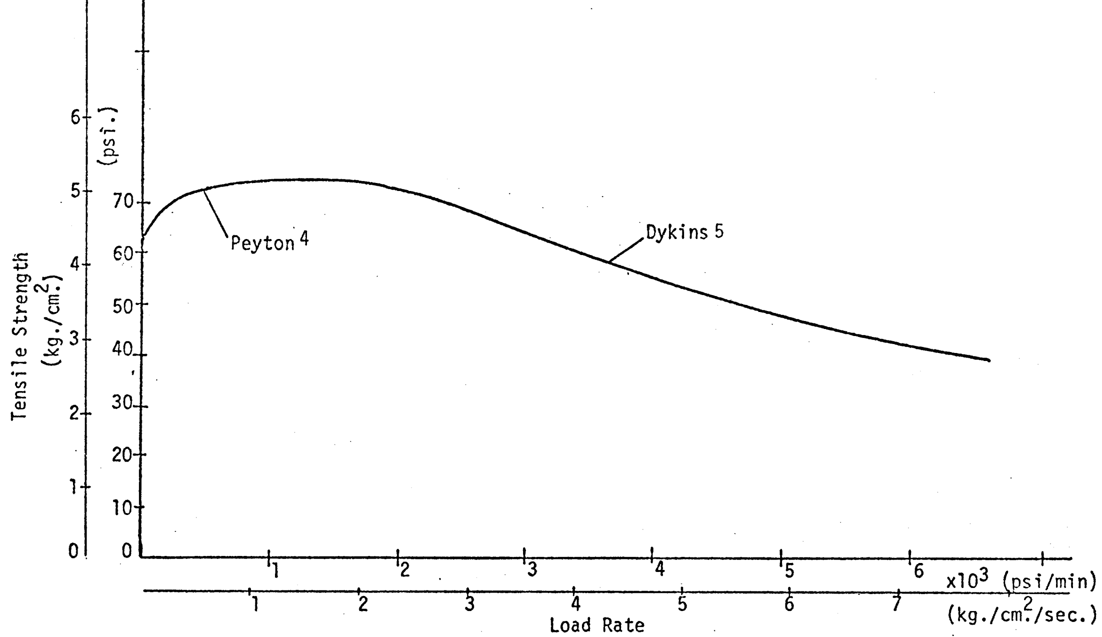


Figure 6 Tensile Strength vs. Load Rate-Sea Ice (arbitrary reference point)

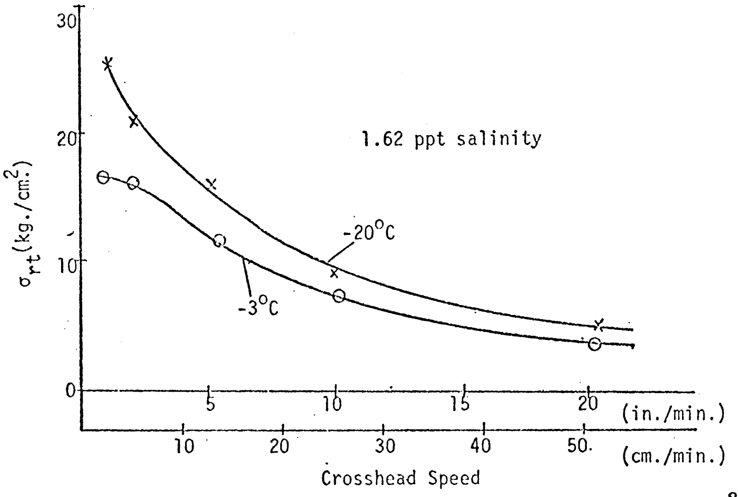


Figure 8 Ring Tensile Strength vs. Crosshead Speed [8]

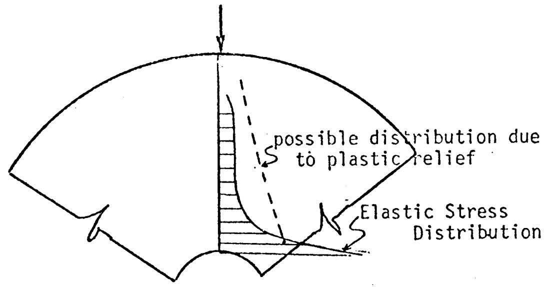


Figure 9 Tensile Stress Distribution in a Ring Tensile Test

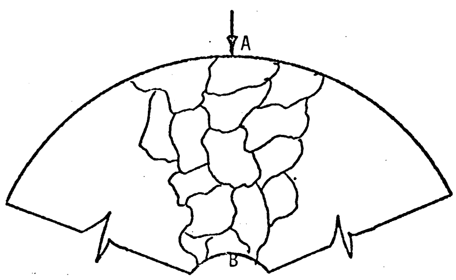
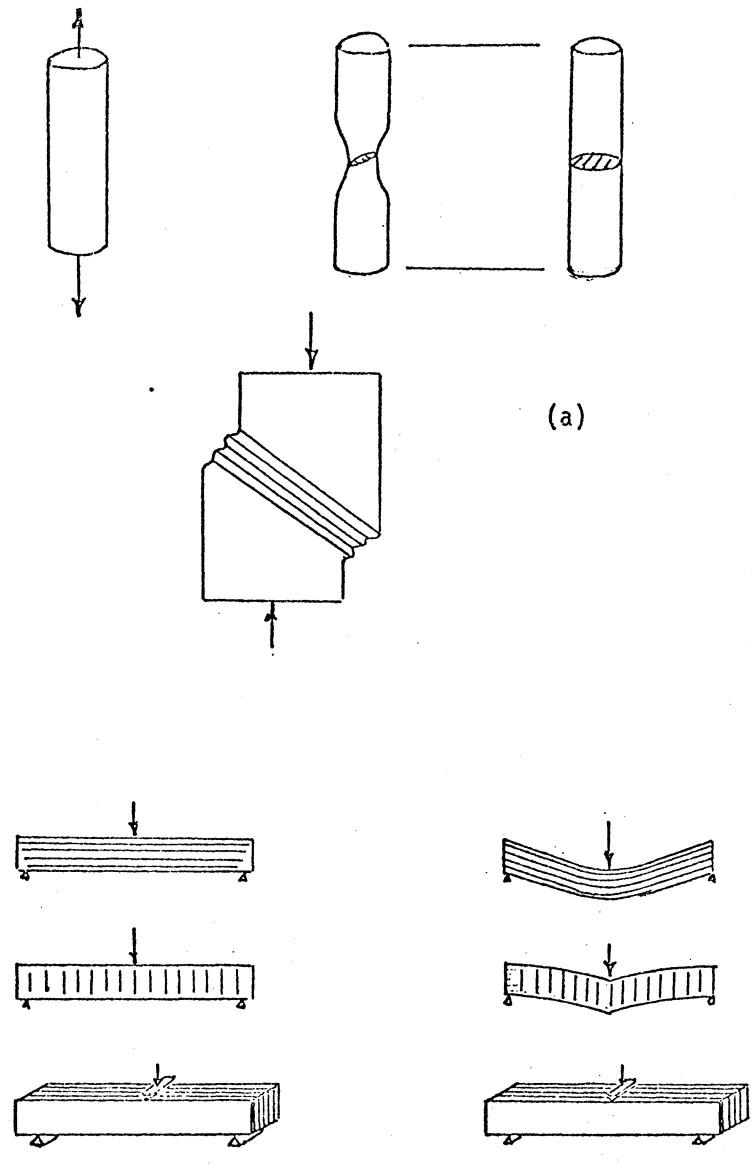
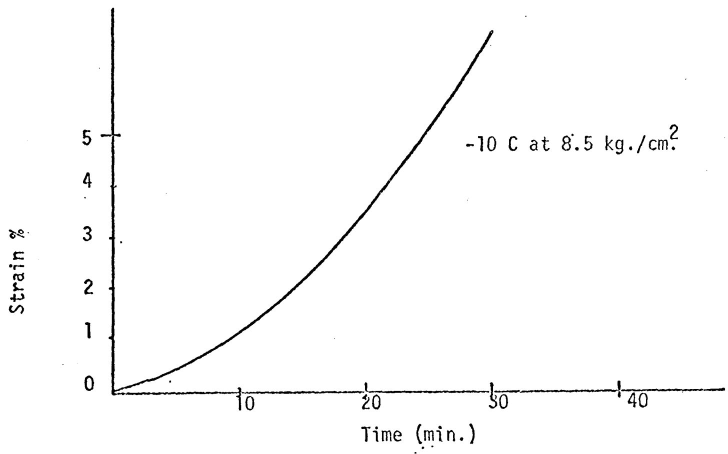


Figure 10 Grain Structure Under Ring Tensile Load

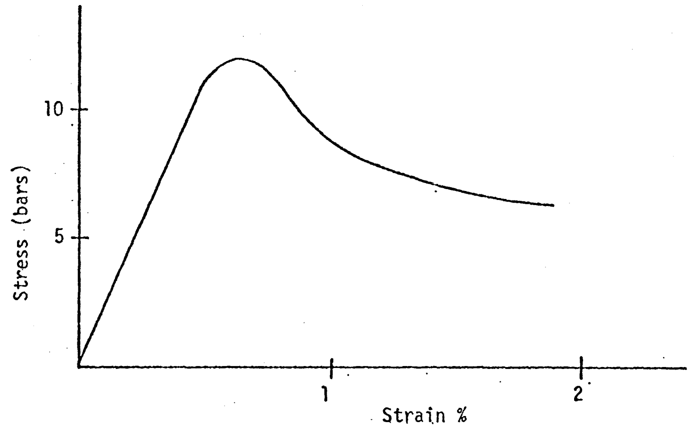


(b) (from Nakaya[16])

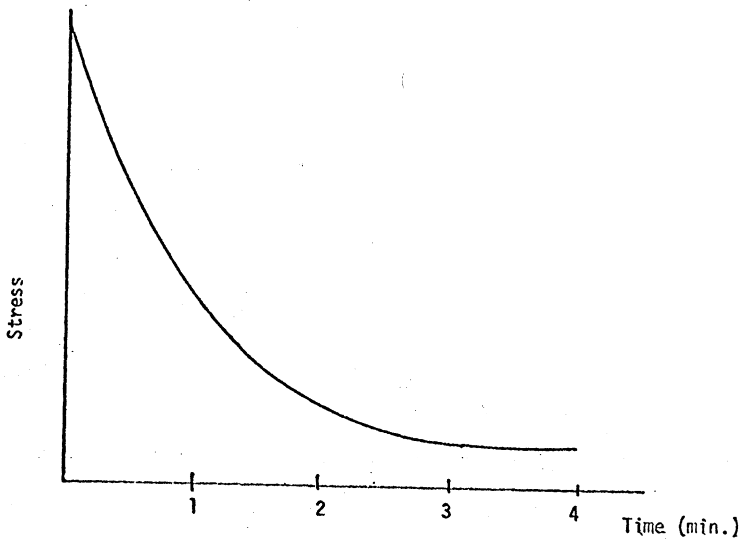
Figure 11 Geometry of Single Crystal Deformation



Typical Creep Curve [12]



Stress-Strain Curve at Constant Crosshead Speed [17]



Relaxation Curve [18]

Figure 12a Time-Dependent Plasticity of Single Crystals of Ice

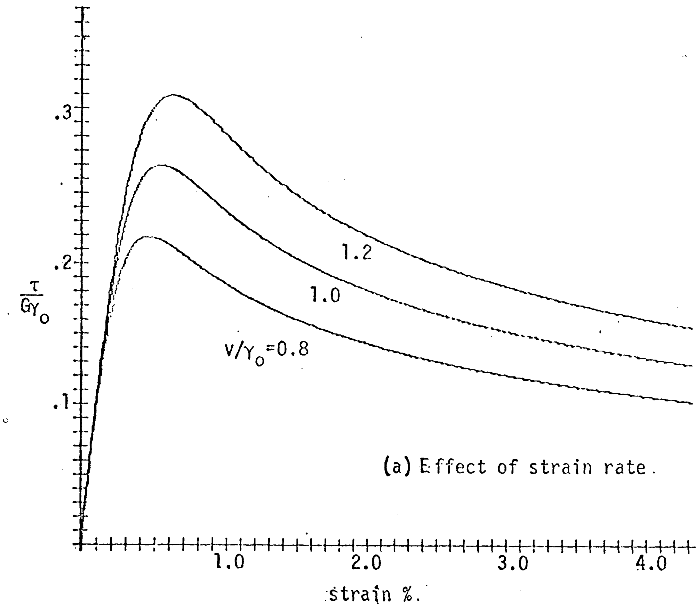
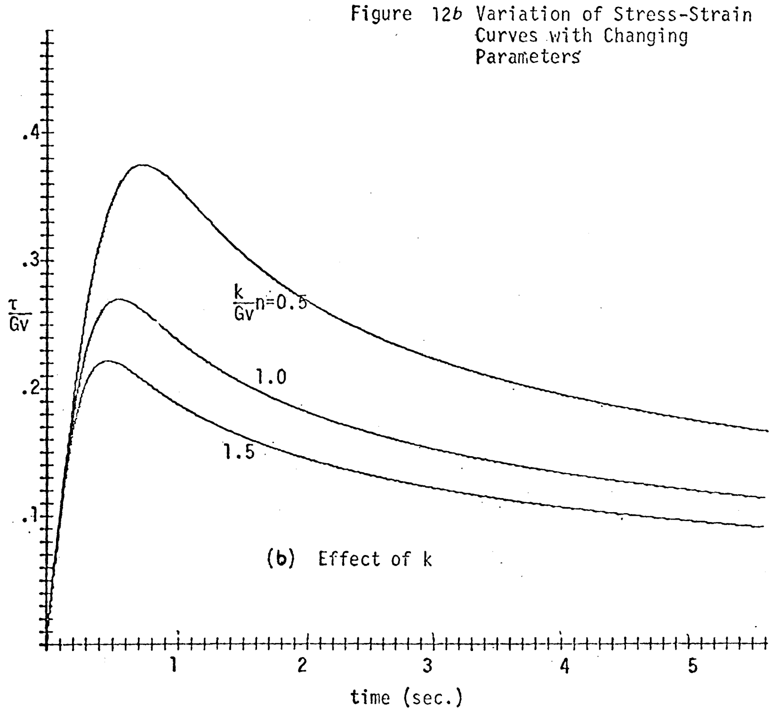
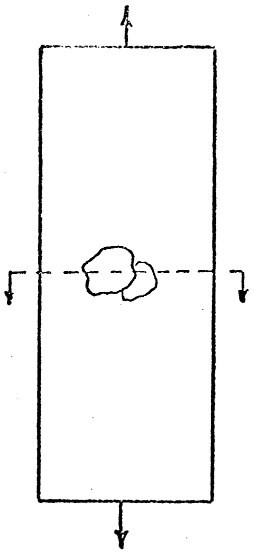


Figure 12b Variation of Stress-Strain Curves with Changing Parameters





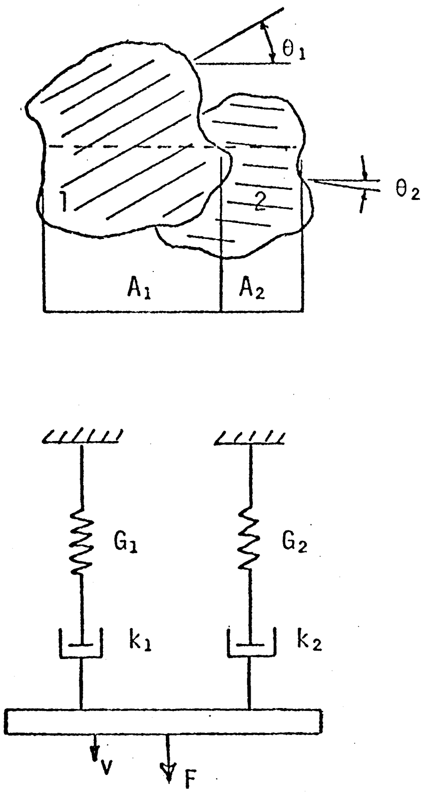


Figure 13 A Two-Grain Model

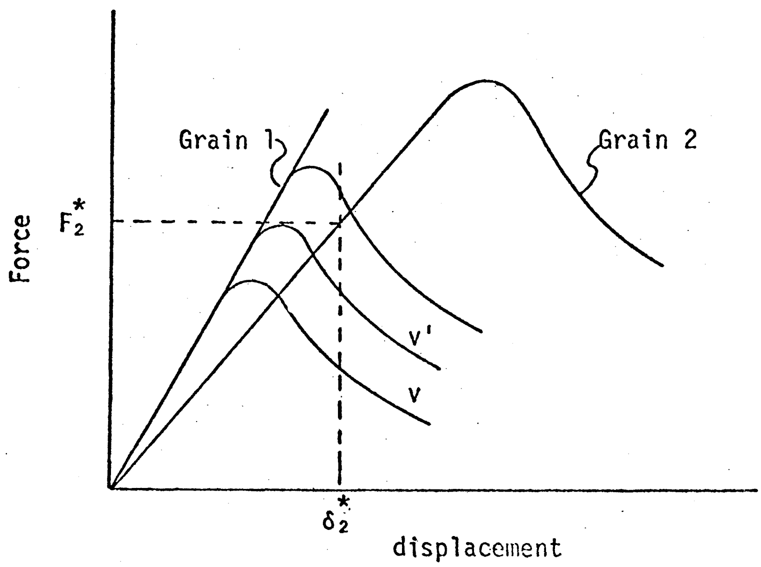


Figure 14 Spring-Dashpot Force-Displacement Curves

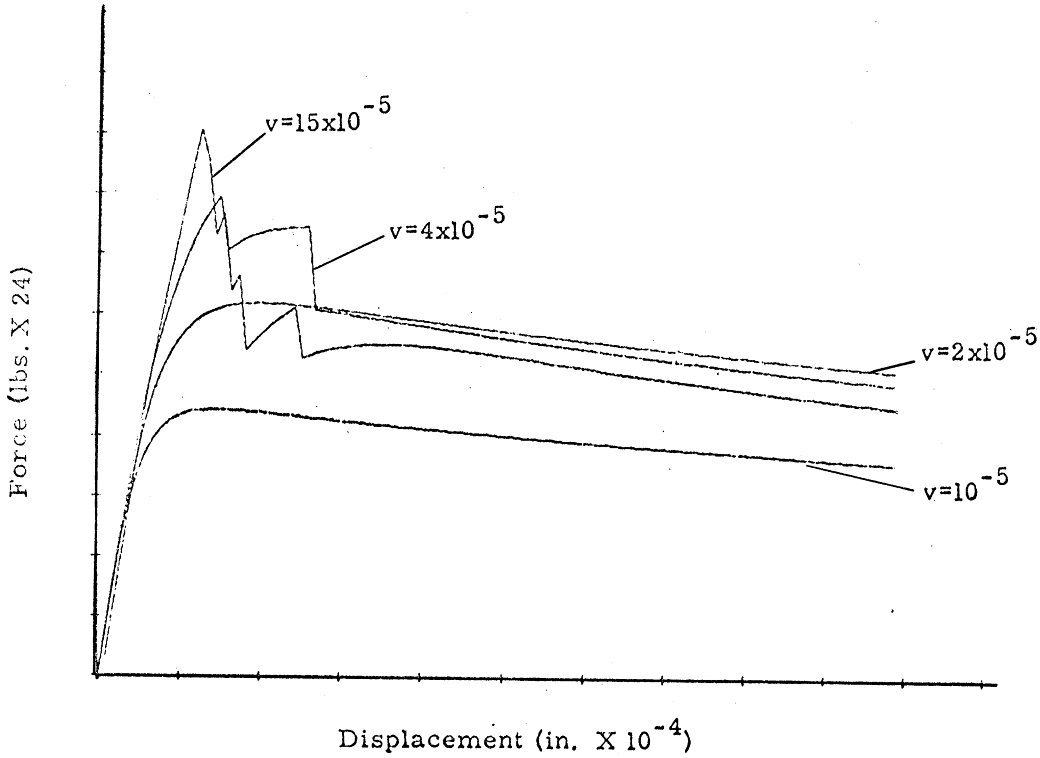


Figure 15

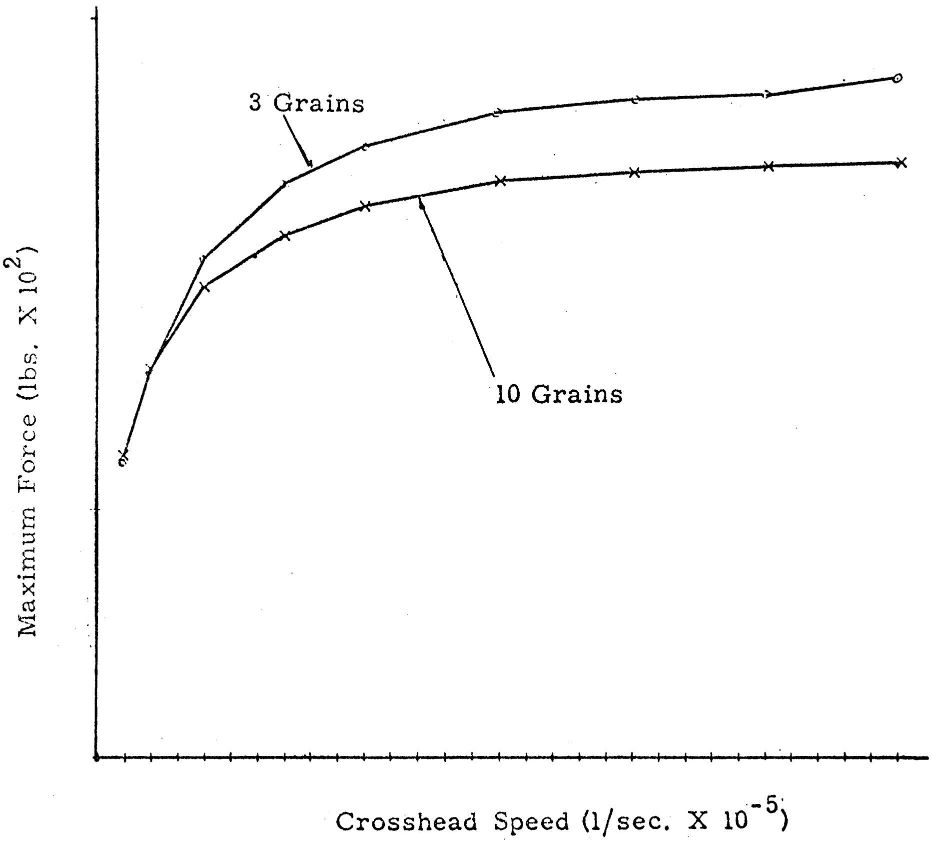


Figure 16

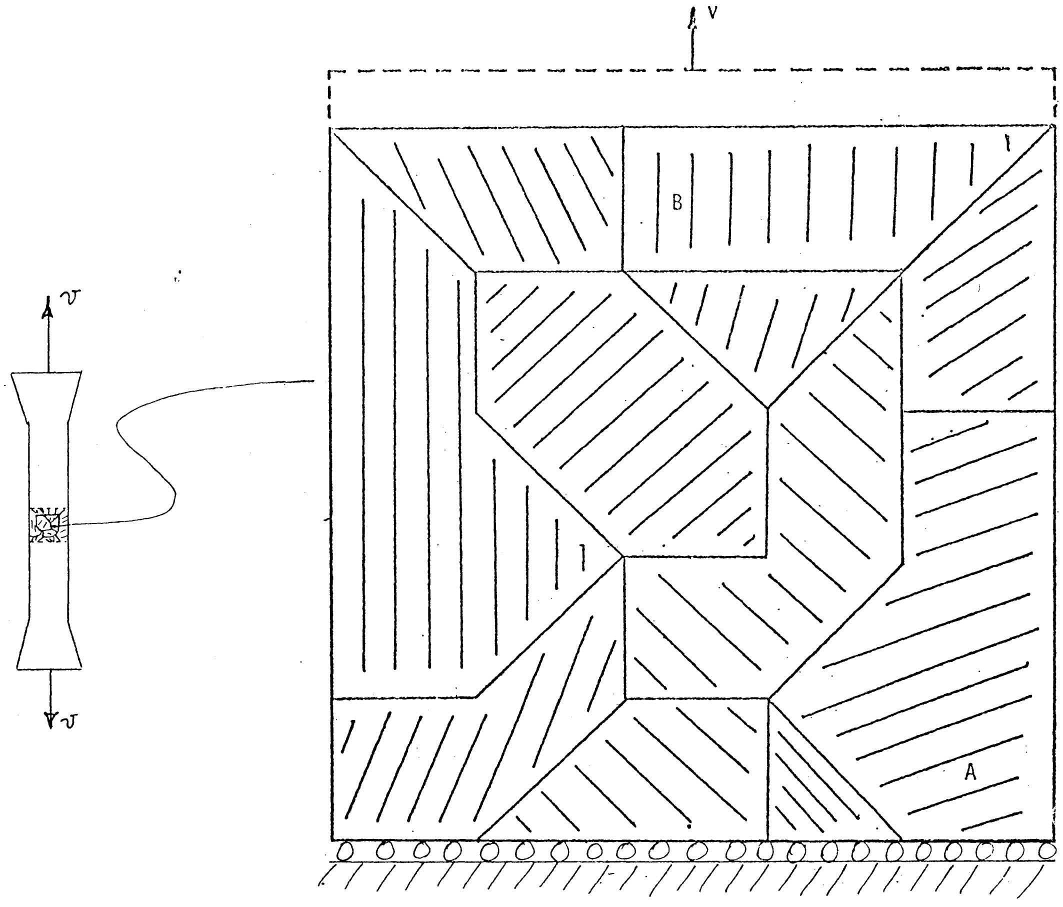


Figure 17 Finite Element Simulation of a Uniaxial Tensile Test

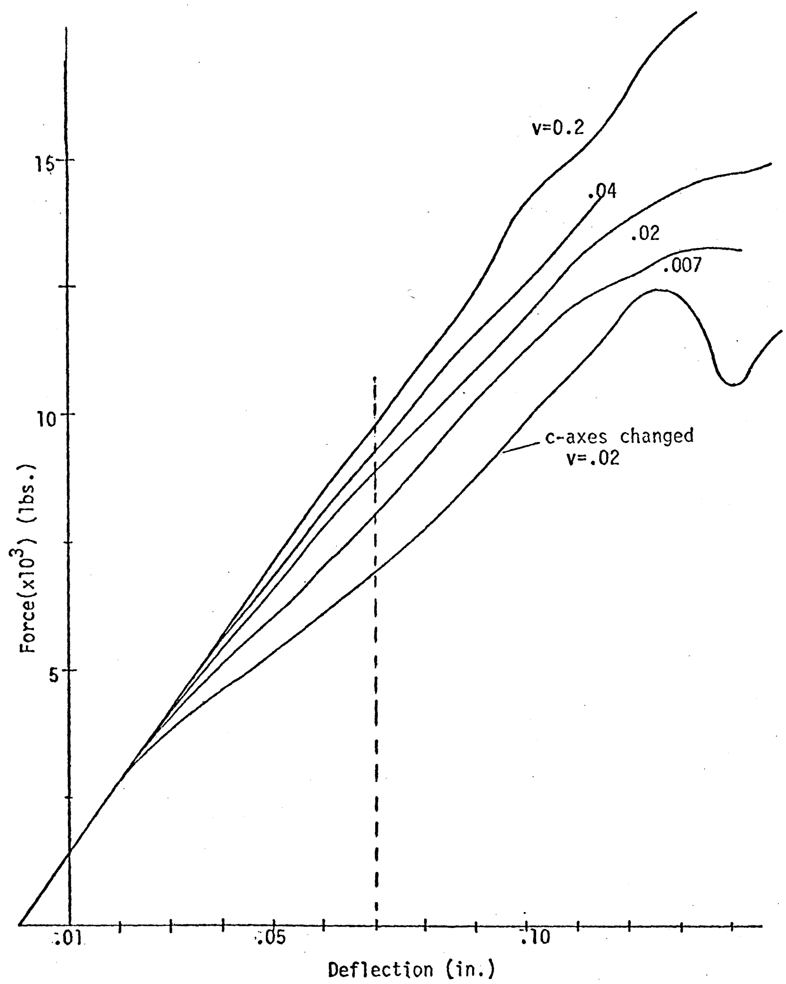


Figure 18 Simulated Force-Deflection Curves for Varying Strain Rates

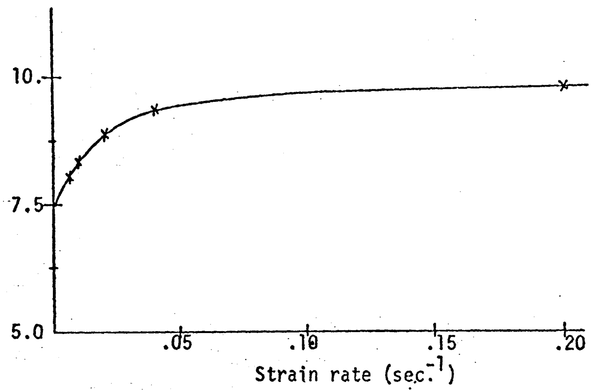


Figure 19 Failure Load vs. Strain Rate