PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

ON THE DURATION OF STORMS IN THE NORTH SEA

Ole G. Houmb

Associate Research Engineer, Technical University of Norway, Trondheim Norway

The occurrence of storms and their duration have been investigated for two locations in the North Sea. The duration of a storm is defined as the time when the wave height equals or exceeds 2.5 meters. It is found that the number of storms per unit time according to this definition is described by the Poisson distribution, and that the duration of the storms equally well follows the exponential distribution.

1. Introduction

The engineering literature frequently contains papers on long term and extreme value distributions of wave heights. Also the probabilities of occurrence of extreme wave heights have been investigated by serveral authors applying the Poisson distribution. However, little attention has been paid to the duration of storms as well as the duration of intervals between storms. These parameters are important for the planning and the economy of offshore operations.

According to the author's information, the most important work regarding durations of this kind has been done by A. Jensen et.al.(1) and P.J. Rijkoort and J. Hemelrijk (2). Jensen et.al. investigated risks of damage of Danish dikes due to storm surges.

Rijkoort and Hemelrijk investigated whether twin storms occur more often than expected under the hypothesis of randomness. The problem may also be stated in the following way: is the probability of occurrence of a storm raised if another storm has occurred a few days before? They concluded that twin storms occur more often than expected by assuming storms to be generated by a stochastic process.

The investigation herein yields the occurrence and the duration of storms only. As the definition of a storm is in terms of a certain wave intensity, the analyses are based on wave data. The data are visually estimated wave heights at the lighthouse of Utsira and the weather and rescue ship Famita in the North Sea, Fig. 1.

A simple Poisson model forms the theoretical basis for the study.

2. The statistical model

A Poisson process is a stochastic process describing point events occurring singly in time according to probabilistic laws. The properties defining a Poisson process are as follows: Let $N(t,t+\Delta t)$ denote the number of events in the interval $(t,t+\Delta t)$ on the real axis. Assume that as $\Delta t \to 0$, we have a constant λ, $\lambda>0$ so that

$$\text{prob}|N(t,t+\Delta t) = 0| = 1 - \lambda(\Delta t) + \rho(\Delta t) \tag{1}$$

$$\text{prob}|N(t,t+\Delta t) = 1| = \lambda(\Delta t) + \rho(\Delta t) \tag{2}$$

and consequently

$$\text{prob}|N(t,t+\Delta t) > 1| = \rho(\Delta t) \tag{3}$$

where $\rho(\Delta t)$ is a function tending to zero more rapidly than Δt. $N(t,t+\Delta t)$ is independent of occurrences in $(0,t)$, which specifies the randomness of the process. Eq. (3) specifies that the events occur singly in time.

The counting distribution of a Poisson process is the Poisson distribution which states

$$p|N(t) = n| = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \tag{4}$$

with mean and variance given by

$$E|N(t)| = \text{Var}|N(t)| = \lambda t \tag{5}$$

λ being the average number of events per unit time, or the intensity of the process.

One of the elementary properties of the Poisson process is that the probability distribution of distances τ between consecutive events is exponential having the density distribution

$$f_\tau(t) = \frac{1}{\lambda} e^{-\frac{1}{\lambda}t} \tag{6}$$

with mean and variance given by

$$E|\tau| = \frac{1}{\lambda} \; ; \quad \text{Var}|\tau| = \frac{1}{\lambda^2} \tag{7}$$

Parzen (3)

The following definition of a storm has been adopted: A period of time within which the visually observed wave height $H_V \geq 2.5$ meters. If in a series of observations at 6 hr. intervals, only one falls below 2.5 meters, the storm duration is not considered interupted. When the word storm is mentioned later, it is referred to this definition. No difference is made between wind sea and swell, so the storm may be caused by wind sea, or swell, or even mixed wind sea and swell.

Fig. 2 shows the time history of H_V plotted as observations at 6 hr. intervals. The Poisson points may be represented by the crossings of the level $H_V = 2.5$ meters. Two different Poisson processes are present; no.I representing the upcrossings and no. II the downcrossings of the level 2.5 meters. Points a and c are generated by process I, and points b and f by process II. According to the above mentioned definition of a storm, points d and e are not taken as crossings of the level 2.5 meters because in each case one observation only falls below the limiting level.

For Poisson process no. I, which represents the upcrossings, the waiting time for a new storm starts at points b and f, because no new storm can occur while one is already present. The waiting time in this process is then equal to the time between the storms. In agreement with the properties of a Poisson process as mentioned before, it follows that the time distances between storms are exponentially distributed.

For Poisson process no. II represented by the downcrossings of the level 2.5 meters, the waiting time for a new period of $H_V < 2.5$ meters starts at the upcrossings, that is at points a and c. This is because no new "calm" period can occur when one is present. The waiting time for this process is therefore analogous to the duration of the storms which then also is exponentially distributed.

The conclusion so far is therefore: On a pure theoretical basis the duration of storms as well as the time distance between them are exponentially distributed.

One reservation has to be made regarding the time distance between storms. Rijkoort and Hemelrijk concluded in their investigation (2) that twin storms according to their definition occur more frequently than expected under the hypothesis of randomness. It is however, not fully evident that the results of Rijkoort and Hemelrijk are applicable to this particular investigation because:

i) the definition of a storm as assumed in this paper is different from Rijkoort and Hemelrijk's definition, and the data may therefore come from different populations.

ii) Rijkoort and Hemelrijk considered northwesterly storms only, and a slightly different conclusion might have been reached if storms from all directions were taken into account.

It is stressed that there is no physical evidence for remarks i) and ii).

3. The data

Visual data obtained by rescue and weather ship Famita and at the lighthouse of Utsira, Fig. 1, form the basis for the present investigation. The Utsira data include daily observations of meteorological parameters and wave heights for the 20 year period from 1949 to 1968, whereas the observations at Famita are undertaken 8 times per day covering the 11 years from 1959 to 1969. Further the observational routine at Famita comprises wave height reported in half meter intervals for the months October to March. At Utsira the wave height is reported in intervals as shown in table 1 for all months of the year.

Objections may be raised against using such visual data, and especially the lighthouse data which are of relatively low resolution for the examination of storm duration. Several authors, however, conclude (4), (5), (6), that in the lower ranges there is a good agreement between visually and instrumentally obtained wave data. As the assumed definition of storm duration refers to a wave height of 2.5 meters, it is concluded that the visual data form an appropriate base for the study. The alternative approach is to apply instrumentally recorded data. Unfortunately measured wave data of a required time length do not exist, and consequently one is forced to utilize what is available.

4. Analysis of data

Using the chi-square test, the hypothesis that the storms are sampled from a population having a Poisson distribution, was tested. The test result proved this to be correct within the 5 percent level of dignificance, and consequently there is no reason to reject the hypothesis that the storms are generated by a Poisson process.

The hypothesis that the duration of storms is exponentially distributed was also tested applying the chi-square test. This hypothesis was likewise shown to be correct within the 5% level of significance.

The duration of wind force $\geq$ 25 kts at Utsira was also found to be exponentially distributed within the 5% level of significance.

The investigations of the Utsira data are carried out for all months except for June and July. Because Famita is on the site from October to March, the investigations of storm duration for these data cover only 6 months.

The duration data are plotted on semilogarithmic paper applying risk criteria as outlined by Wemelsfelder (7), which is briefly outlined below.

Recalling the Poisson distribution

$$p|N(t) = n| = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

For n = 0 we get

$$p|N(t) = n| = e^{-\lambda t} \qquad (8)$$

which is the probability that the number of events corresponding to λ will not be exceeded in t years. $m = \lambda t = 1$ means that t equals the return period. Substituting m = 1 in eq. (8) gives p = 0.37, meaning that the probability of exceedance of the number of events given by λ is 0.63. For m = 5, 0.1 and 0.01 we get the probabilities of exceedance 99.3, 10 and 5% respectively.

Taking this into account, straight lines representing the probabilities of exceedance 99.3, 63, 10 and 5% may be introduced, the line of 63% being given by the plotting positions.

Figs. 3 to 5 show plottings of the data according to the previously outlined method. The ordinate gives the duration in hours, and the abscissa gives the number of exceedances per year for the respective month.

In order to give a visual impression of the goodness of fit, figs. 3 to 8 include typical good and poor fits of storm durations at Famita and Utsira and the duration of wind force $\geq$ 25 kts at Utsira.

Fig. 9 is a plot of storm durations at Utsira and Famita for all months covered by the data and for the yearly frequencies 0.01, 0.1, 1 and 5. It is evident that the durations are longer at Utsira than those at Famita, which is most probably due to the fact that Utsira is more exposed to swell.

5. Conclusion

Wind and visual wave data obtained at the lighthouse of Utsira and visual wave data from the weather and rescue ship Famita have been investigated.

The hypothesis that storms as defined in this paper are generated from a population having a Poisson distribution is found, by means of the chi-square test, to be correct within the 5% level of significance.

One consequence of this finding, which in theory, is that storm durations are exponentially distributed. This has also been proved by the chi-square test to be accurate within the 5% level of significance.

The same level of significance also applies for the duration of wind force $\geq$ 25 kts at Utsira.

It is evident that storm durations are longer at Utsira than they are at Famita. Most probably this is due to swell from the North Atlantic Ocean and the Norwegian Sea which is more frequent at Utsira.

REFERENCES

(1) Jensen,,A. et al. Analyse af stormflodsrisiko. Report on storm surges. Technical University of Denmark. Copenhagen 1967.

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(4) Nordenstrøm, N. and Pedersen, B. Influence of the relationship between visual and instrumental wave height and period on calculated long term distributions. Report No. 66-31-5, Det norske Veritas.

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(6) Warnsinch, W.H. et al. Report Committee 1. International Ship Structures Congress, Delft 1964.

(7) Wemelfelder, P.J. On the use of frequency curves of stormfloods. Coastal Engineering, vol. 2 (1961).

Class number	Wave height intervals in meters
0	0
1	0 - 0.1
2	0.1 - 0.5
3	0.5 - 1.25
4	1.25 - 2.5
5	2.5 - 4
6	4 - 6
7	6 - 9
8	9 - 14
9	> 14

Table 1. Visual wave height scale in use at Norwegian lighthouses.

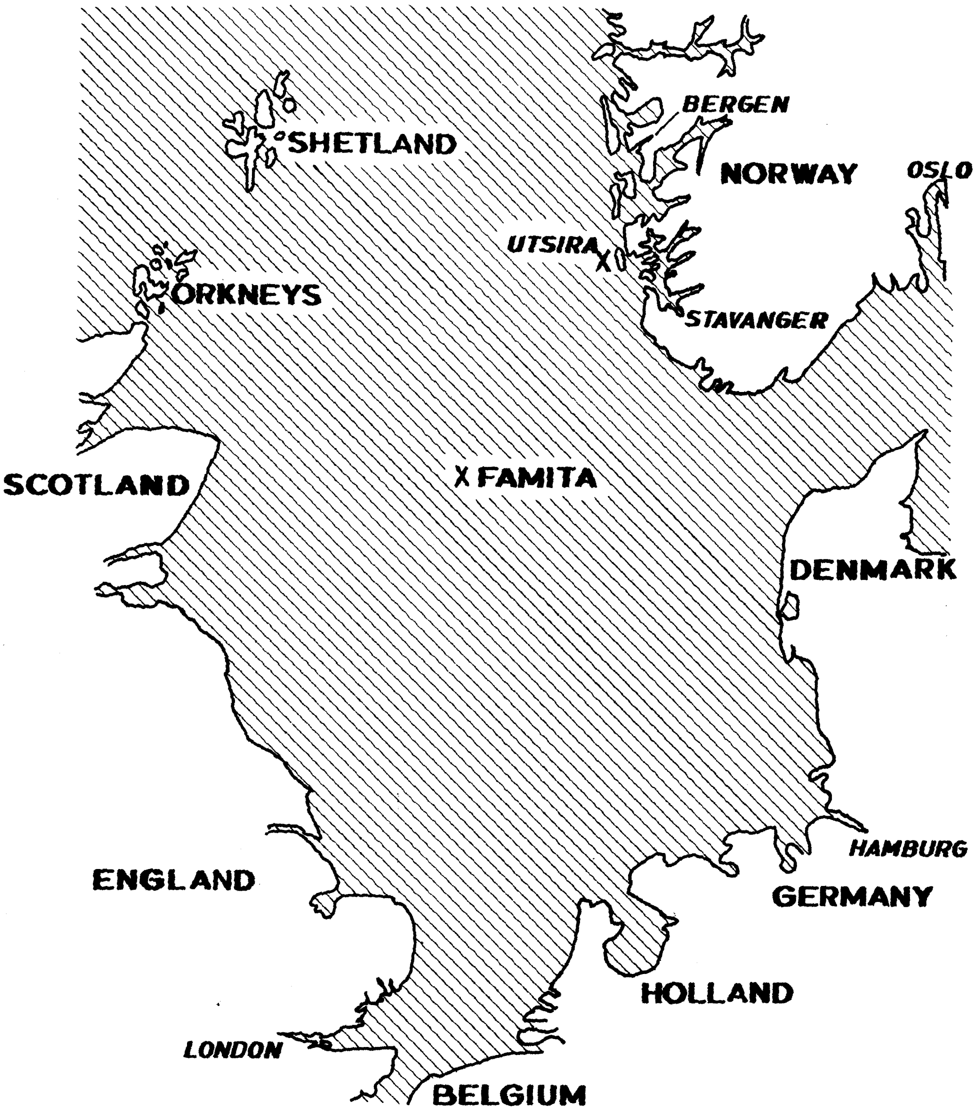

FIG. 1. THE NORTH SEA

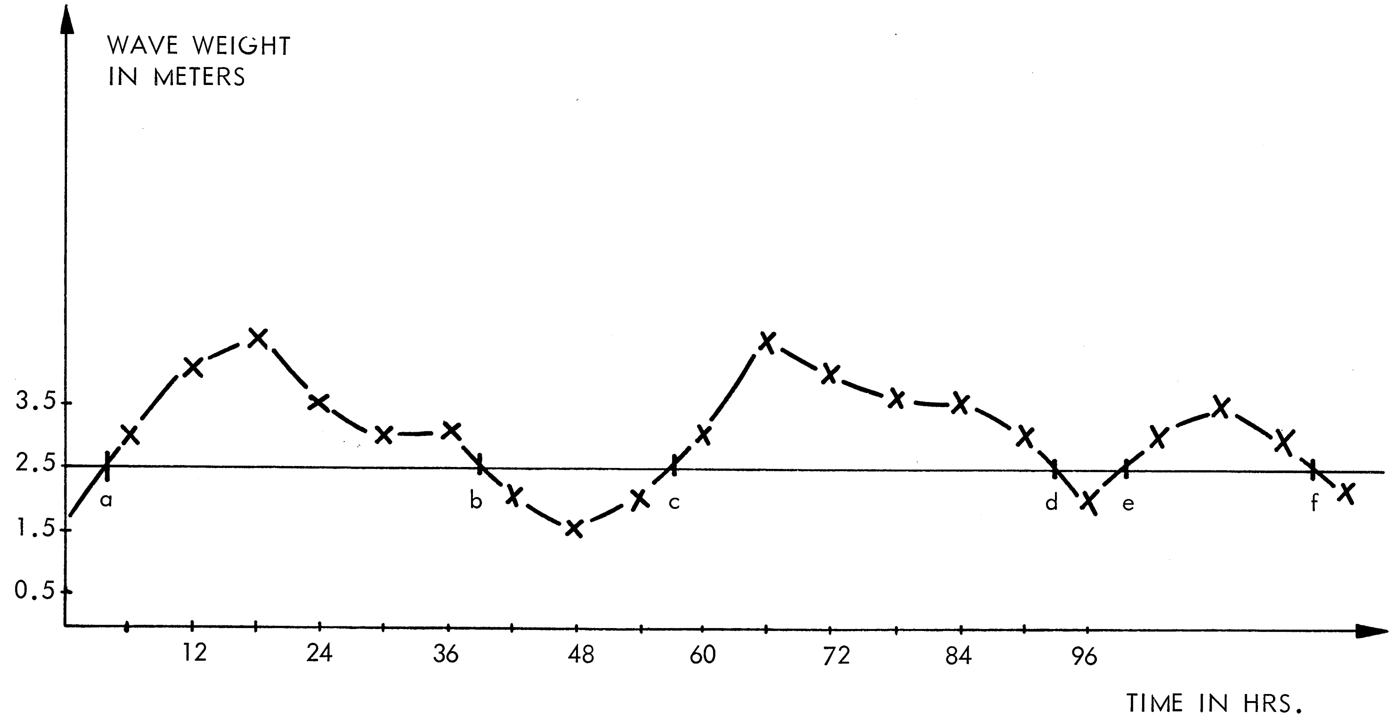

FIG. 2. TIME HISTORY OF H_v

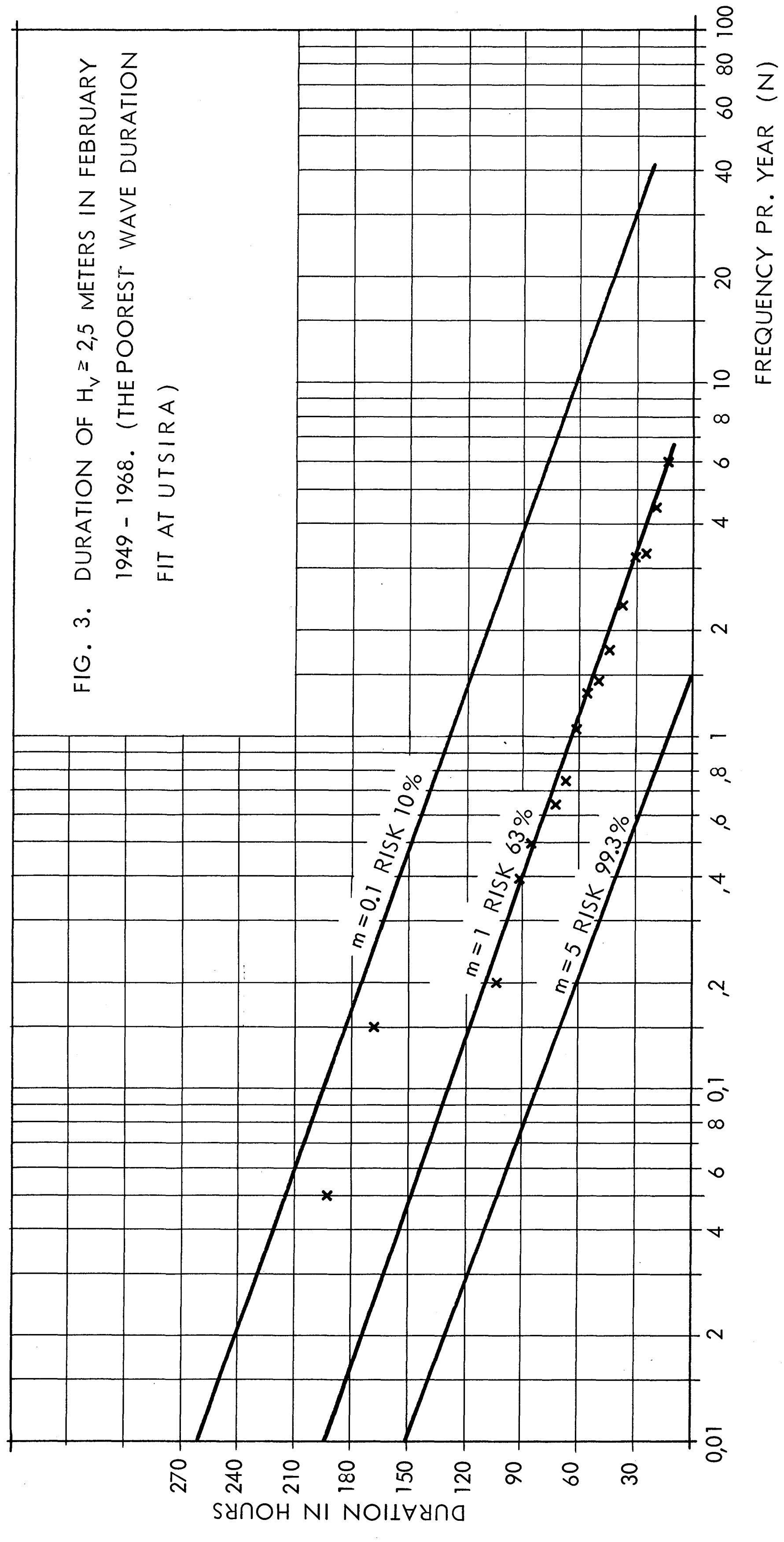

FIG. 3. DURATION OF $H_v \geq 2{,}5$ METERS IN FEBRUARY 1949 - 1968. (THE POOREST WAVE DURATION FIT AT UTSIRA)

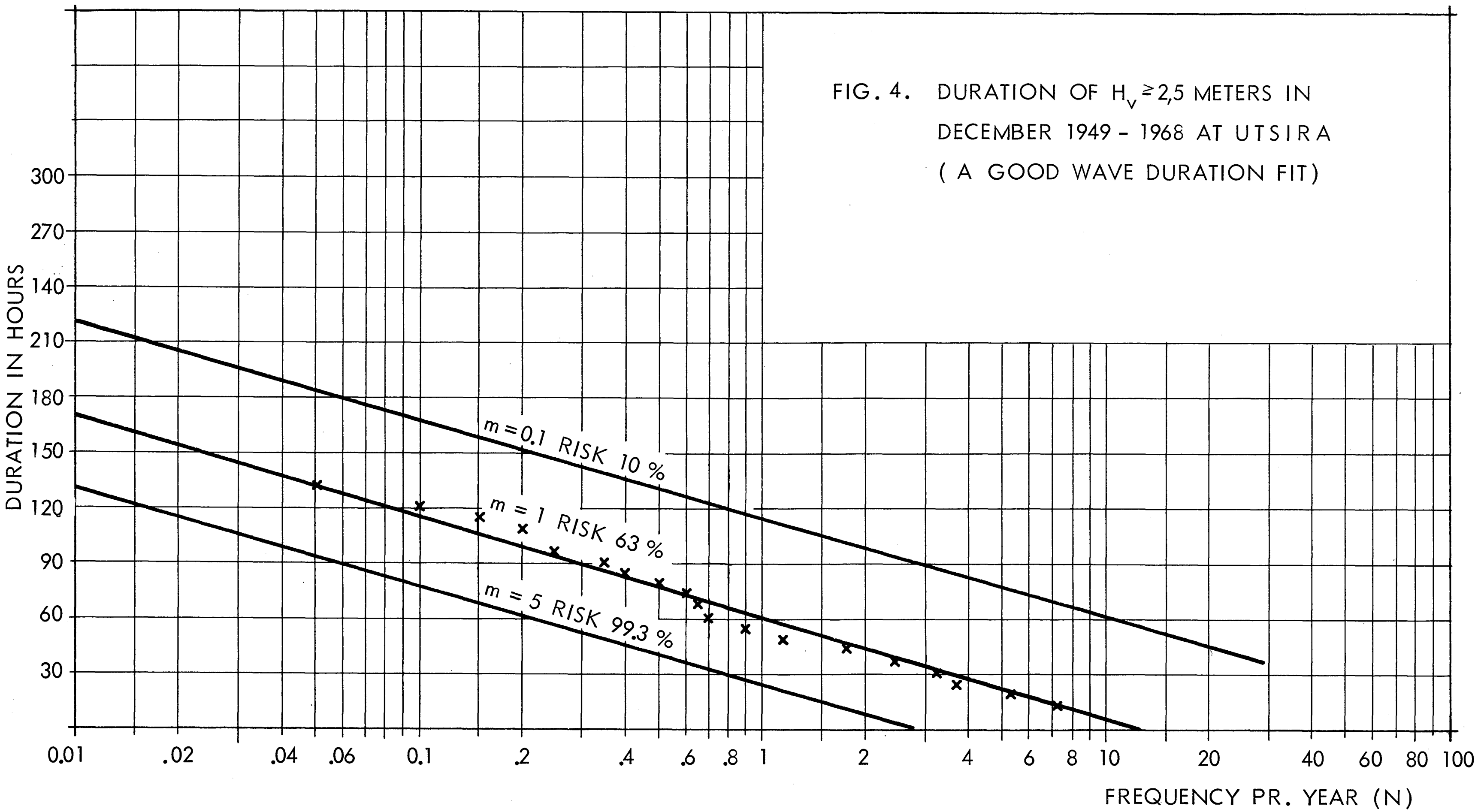

FIG. 4. DURATION OF $H_V \geq 2{,}5$ METERS IN DECEMBER 1949 - 1968 AT UTSIRA (A GOOD WAVE DURATION FIT)

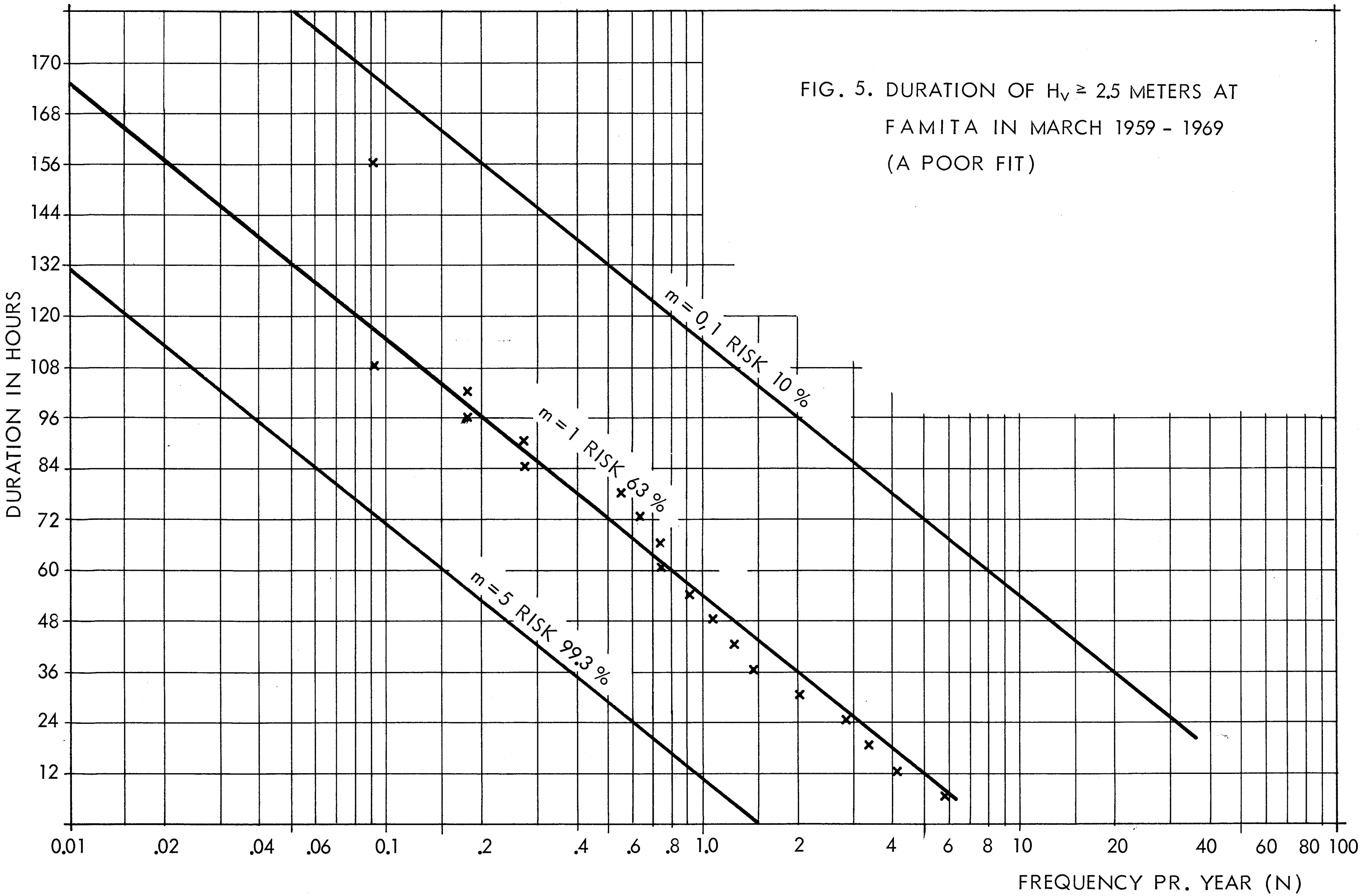

FIG. 5. DURATION OF $H_V \geq 2.5$ METERS AT FAMITA IN MARCH 1959 - 1969 (A POOR FIT)

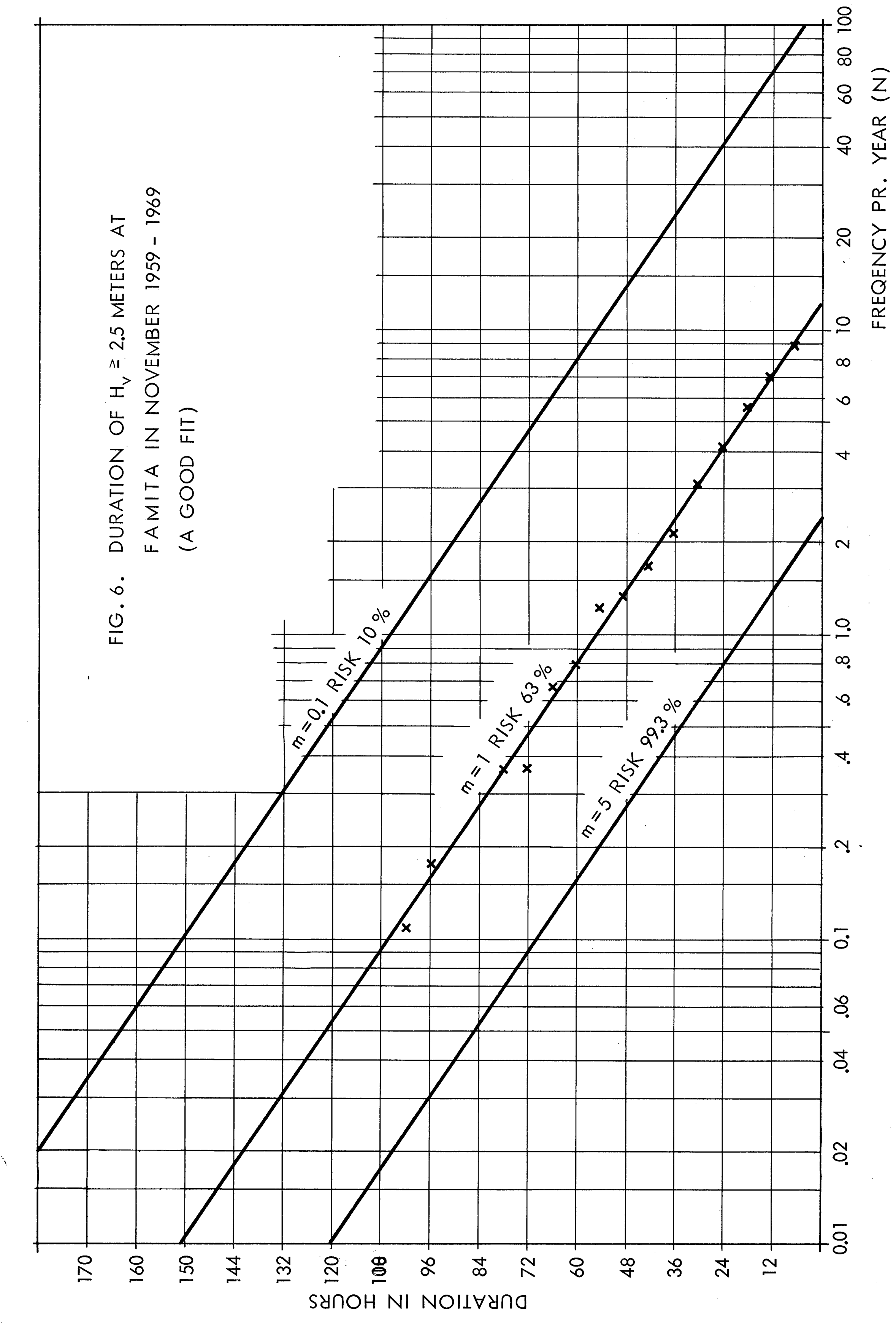

FIG. 6. DURATION OF $H_v \geq 2.5$ METERS AT FAMITA IN NOVEMBER 1959 - 1969 (A GOOD FIT)

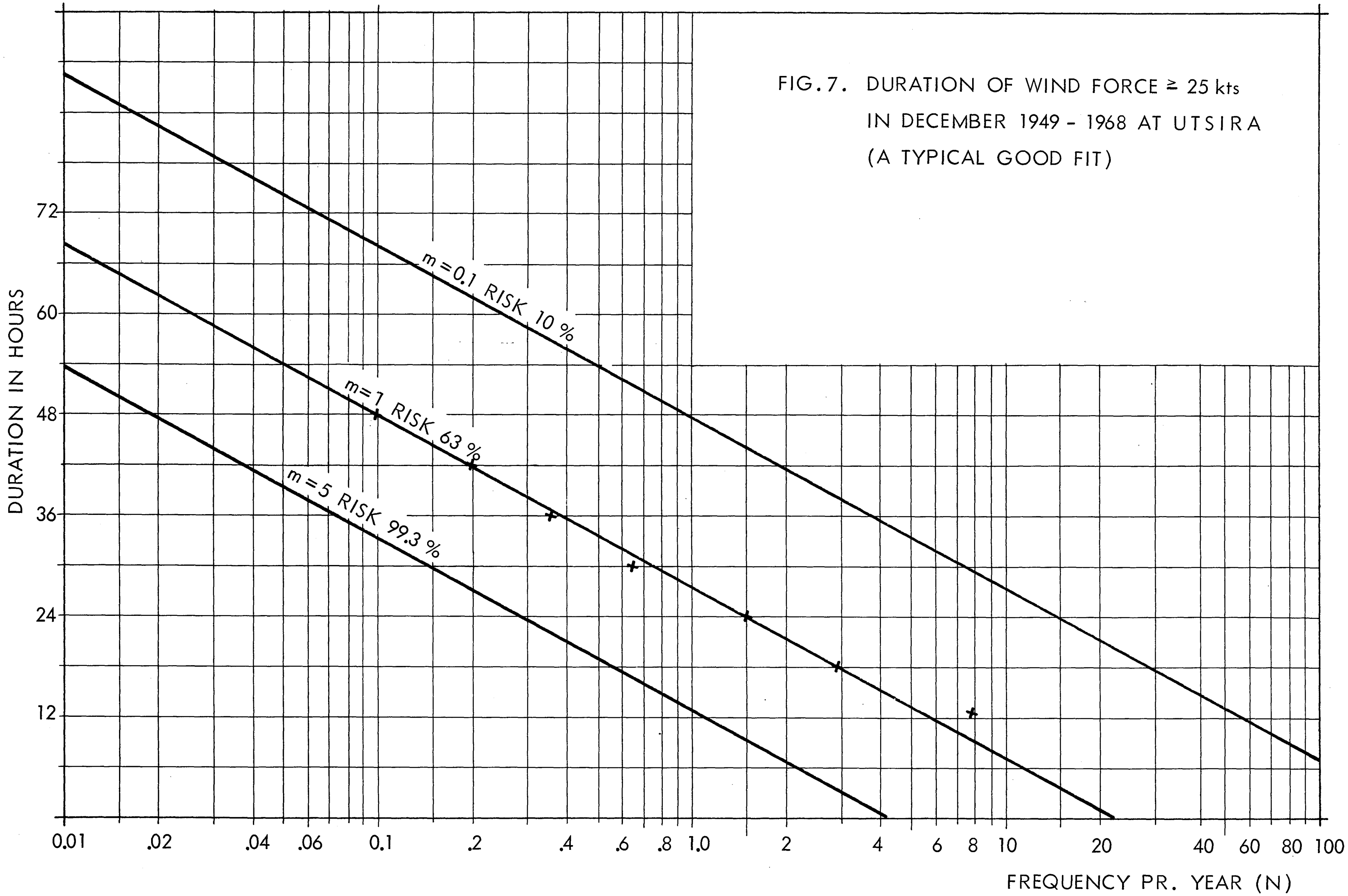
FIG. 7. DURATION OF WIND FORCE ≥ 25 kts
IN DECEMBER 1949 - 1968 AT UTSIRA
(A TYPICAL GOOD FIT)
m = 0.1 RISK 10 %
m = 1 RISK 63 %
m = 5 RISK 99.3 %
DURATION IN HOURS
72
60
48
36
24
12
0.01
.02
.04
.06
0.1
.2
.4
.6
.8
1.0
2
4
6
8
10
20
40
60
80
100
FREQUENCY PR. YEAR (N)

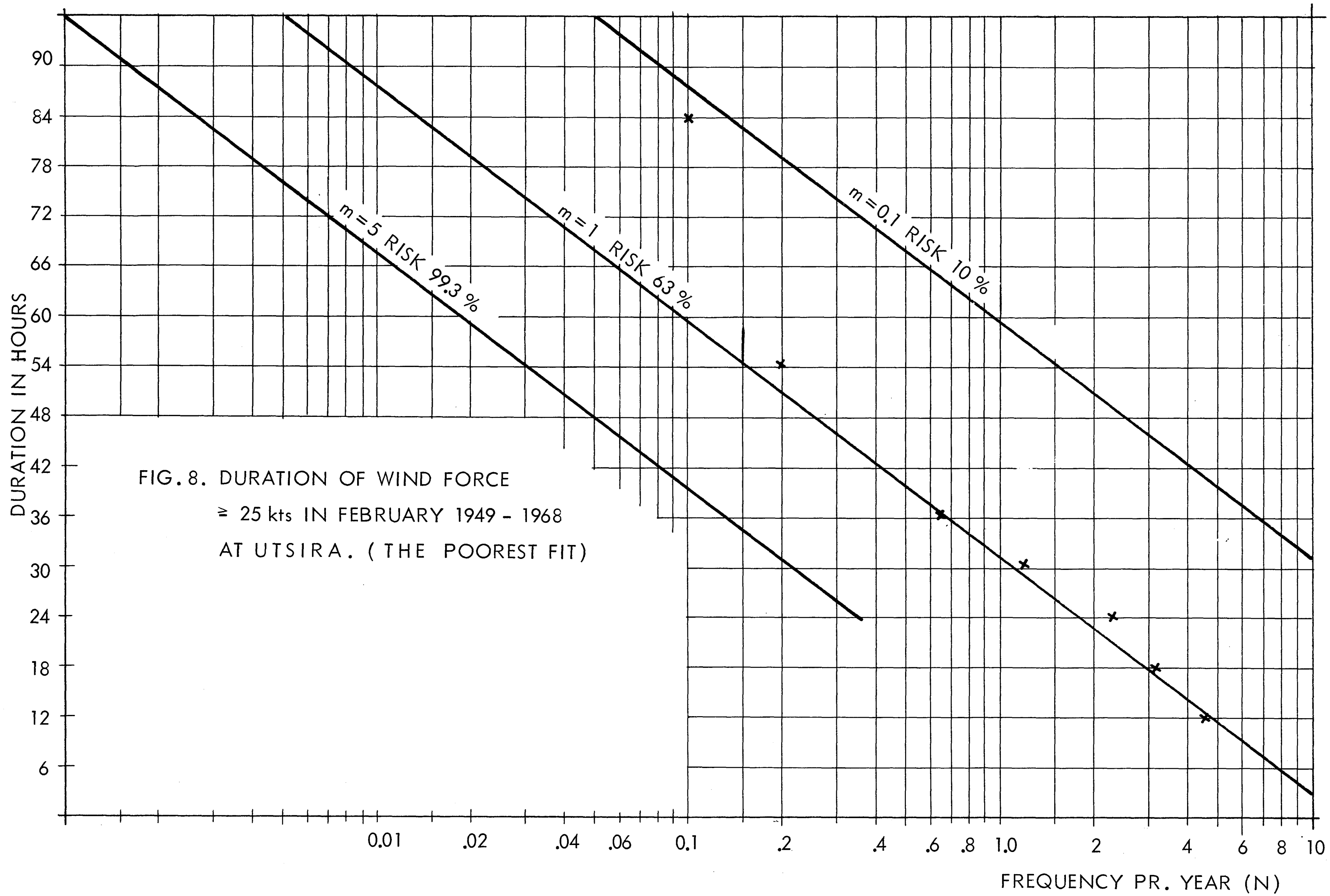

FIG. 8. DURATION OF WIND FORCE
≥ 25 kts IN FEBRUARY 1949 - 1968
AT UTSIRA. (THE POOREST FIT)

FIG. 9. DURATION OF $H_V \geq 2.5$ METERS AT UTSIRA AND FAMITA FOR THE YEARLY FREQUENCIES 5, 1, 0.1 AND 0.01