PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

MECHANICAL PROPERTIES OF SNOW ICE

René O. Ramseier — Research Scientist
Dept. of the Environment, Inland Waters Branch
Ottawa, Ontario, Canada.

ABSTRACT

Most strength values, cited in the literature, for fresh water ice are not well defined in terms of the type of ice or strain rate applied, and therefore their usefulness for engineering purposes is questionable. In this study the uniaxial strength of monocrystals and snow ice was determined over a wide range of strain rates and stresses. The snow ice had equiaxed grains with an average diameter of 0.1 cm and the crystallographic orientation was random.

Two types of tests were used -- tensile and compression. Tensile tests, using constant loads, were carried out to the point where cylindrical samples failed or a secondary creep rate was obtained. This can be considered as a constant stress test since ice has no necking tendencies. Compression tests were performed under constant strain rate conditions. The yield point was taken as the critical value for the analysis. The stress and strain rate behavior was also studied in the temperature range -1.5 to -35C.

The strain rate-stress curve was divided into two regions - the ductile and brittle region. In terms of stress the ductile region was further divided into the low, intermediate, and high stress stages. The latter stage was typified by formation of cracks whereas the intermediate stress stage lacked these characteristics.

The brittle region consisted of a ductile-brittle transition zone which showed a decreasing stress with an increasing strain rate; it eventually became independent of the stress in the quasi-brittle region.

The results now available cover a range of ten orders of magnitude for the strain rate and over two orders of magnitude for the stress. The transition from one zone or region to another depends on temperature.

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INTRODUCTION

The mechanical behavior of crystalline solids at elevated temperatures is primarily controlled by thermally activated processes. These properties are strongly dependent on strain rate and temperature above 0.5 T_m (T_m is the melting temperature in Kelvin). In the case of ice this would mean working in a temperature range of from -136C to 0C. For engineering problems a more useful range is -50C to 0C, or starting at approximately 0.8 T_m.

The two important variables which influence creep are temperature and stress. Therefore, the creep strength of ice can best be studied by a creep test which applies a constant load or stress. Figure 1, a typical creep curve in tension, depicts three stages of creep. The initial increase of the true creep strain from 0 to ϵ_0 is obtained immediately upon loading and can be considered as the amount of elastic deformation. The portion of the creep curve from ϵ_0 to ϵ_1 is known as transient or primary creep. In metals, a stable substructure develops in this stage which is strongly dependent on the creep stress and in a minor way on temperature. Steady state or secondary creep develops from ϵ_1 to ϵ_2. The creep substructure remains practically unchanged in stage II. Finally, the last stage where the creep accelerates from ϵ_2 to ϵ_f is known as tertiary creep. Tertiary creep is frequently accompanied by necking, void formation near grain boundaries, and crack formation leading to a tensile creep failure.

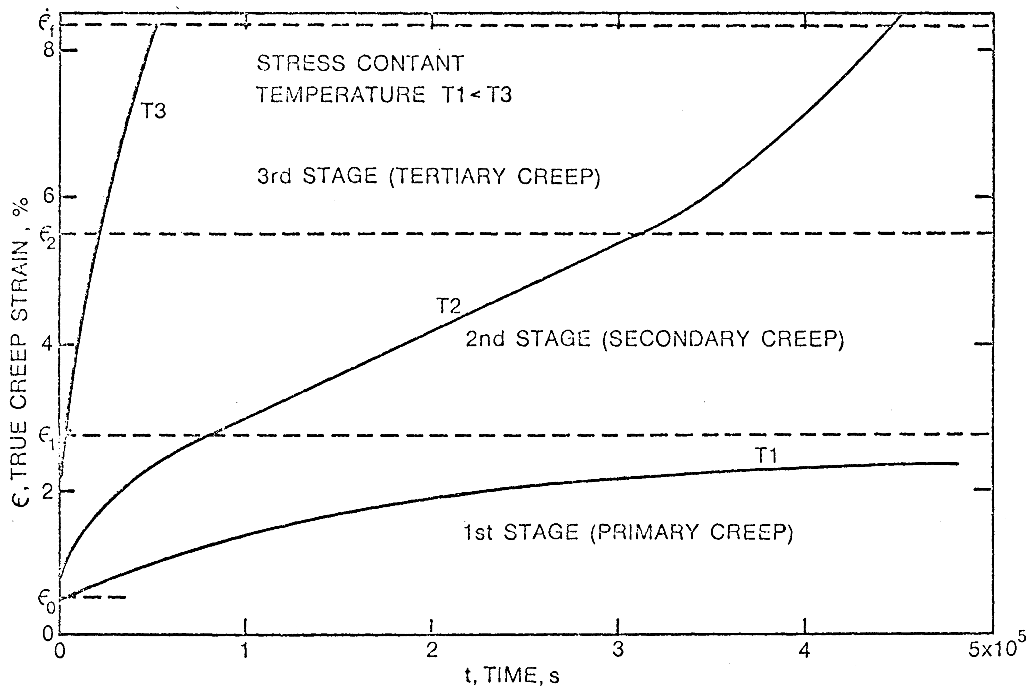

Figure 1 Typical creep curve in tension depicting three stages of creep using a constant load but at different temperatures.

TEMPERATURE DEPENDENCE OF CREEP RATE

A generalized function of the creep rate based on the theory of thermodynamic fluctuations can be given by[1]:

$$\dot{\epsilon} = \sum_i Z_i\,(V,T,S)\,\sigma_i\,(T,S)\exp\left[\frac{-\Delta Q_i(T,S)}{RT}\right] \tag{1}$$

The function Z_i may depend on temperature, molecular vibrational frequency V, and a structure term S. The term S involves features such as grain size, density and dislocations. The stress function, σ_i, depends on temperature and structure term. ΔQ_i is the true activation energy governing the creep rate and is temperature and structure dependent. Results for the creep rate can usually be fitted by simplifying equation 1 to the form:

$$\dot{\epsilon} = K\exp(-Q_c/RT) \tag{2}$$

R is the gas constant, T is the absolute temperature, and K is a constant for a given creep stress.

The activation energy can be calculated if the creep rate is known at two temperatures, T_1 and T_2.

$$Q_C = \frac{R \ln \dot{\epsilon}_1/\dot{\epsilon}_2}{1/T_2 - 1/T_1} \tag{3}$$

Q_c is insensitive to strain creep curves under constant stress, but at different temperatures can be brought into coincidence by the use of a time-temperature parameter of the type t $\exp(-Q_c / RT)$.[2]

DIFFUSION DEPENDENCE OF CREEP RATE

In the secondary region the creep rate should be proportional to the diffusion coefficient at high temperatures as in the form

$$\dot{\epsilon}_s = K D f(\sigma, T) \tag{4}$$

where the diffusion coefficient

$$D = D_0 \exp(-Q_s/RT) \tag{5}$$

It should be obvious then that the activation energy for creep of pure ice should be about equal to the activation energy for self-diffusion. This has been verified for a large number of metals and compounds.[1,3]

INTERMEDIATE STRESS CREEP RANGE

At the intermediate creep stress level the available data can well be represented by a power law function of the type

$$\dot{\epsilon} = K \sigma^n \tag{6}$$

This is not only true for metals but also for ice. However, there is no general agreement on the value of n for ice.

ELASTIC MODULUS

In the theories developed by Weertman[4,5], the creep rate is given as

$$\dot{\epsilon} \propto \left(\frac{\sigma}{E}\right)^n \tag{7}$$

The elastic modulus has been shown to vary with temperature in metals and ice[6]. The activation energy of creep cannot then equal the activation energy of self-diffusion. Based on equations 2 and 4 and in conjunction with equation 7 the steady state creep rate can be described by:

$$\epsilon = KD\left(\frac{\sigma}{E}\right)^{n} \tag{8}$$

The apparent activation energy for creep at constant strain and stress based on equation 3 and with the help of equation 8 is given by:

$$Q_c = -R\,\frac{d\ln\dot{\epsilon}}{d\,l/T} = -R\,\frac{d(\ln D)}{d(l/T)} + nR\,\frac{d(\ln E)}{d(l/T)} \tag{9}$$

The first term is the activation energy for self-diffusion, thus:

$$Q_c = Q_s + nR\,\frac{d(\ln E)}{d(l/T)} \tag{10}$$

or

$$Q_c = Q_s - nR\,\frac{T^2}{E}\,\frac{dE}{dT} \tag{11}$$

Equation 10 predicts that $Q_c > Q_s$ since d(lnE)/d(1T) is positive for ice.

TENSILE CREEP CURVES UNDER CONSTANT LOAD

Figure 2 shows typical creep curves at constant temperatures but three different stresses. The strain dependence of each stage of creep is strongly dependent on stress but insensitive to temperature. Usually the higher the stress, the greater is the strain at the onset of secondary and tertiary creep stages.

Great care was taken to insure that a truly secondary creep rate was obtained before unloading the sample to obtain the secondary strain rate dependence at lower stresses. This was necessary to shorten the test period by weeks and months.

To check the effect of substructure, two tests were performed at constant temperature but different stresses. A sample was subjected to a stress of 3.84 $kg.cm^{-2}$ and maintained at this stress throughout the test. Another sample was subjected to a higher stress and then unloaded to the same stress as the first sample. The strain rates were almost equal, indicating that for the particular stresses used the effect of substructure is negligible.[7]

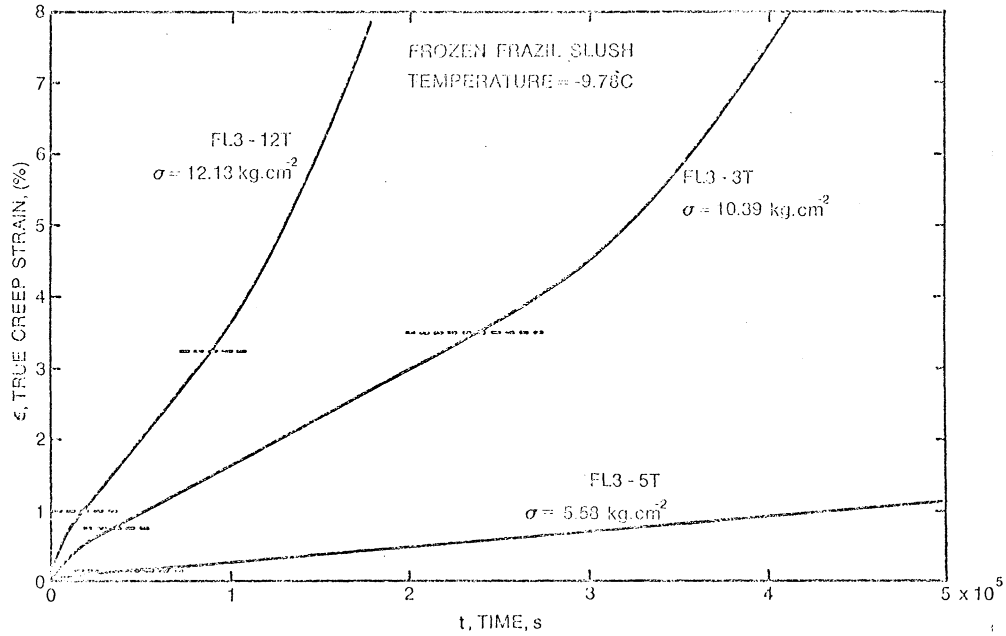

Figure 2 Tensile creep curve under constant load.

CONSTANT STRAIN RATE CREEP CURVES

These curves fall into two categories:

i - No cracks are observed during the entire test period,

ii - Cracks become visible shortly before the maximum is obtained.

This is illustrated in figure 3. The derivative $\frac{d\sigma}{d\epsilon} = 0$ is of particular interest for the analysis of the curves. Usually the higher the stress, the higher is the strain for the yield point; analogous to the tensile case.

ICE MONOCRYSTAL RESULTS

The results, shown in figure 4, are plotted in a similar manner to the previous results with the exception that the stress is not divided by E, the apparent elastic modulus[7], since it shows no temperature dependence within the measured temperature range of 35C. The fit of the data is very good showing almost no dispersion. Each type of point represents a particular test temperature as explained in the figure caption. To demonstrate the effectiveness of

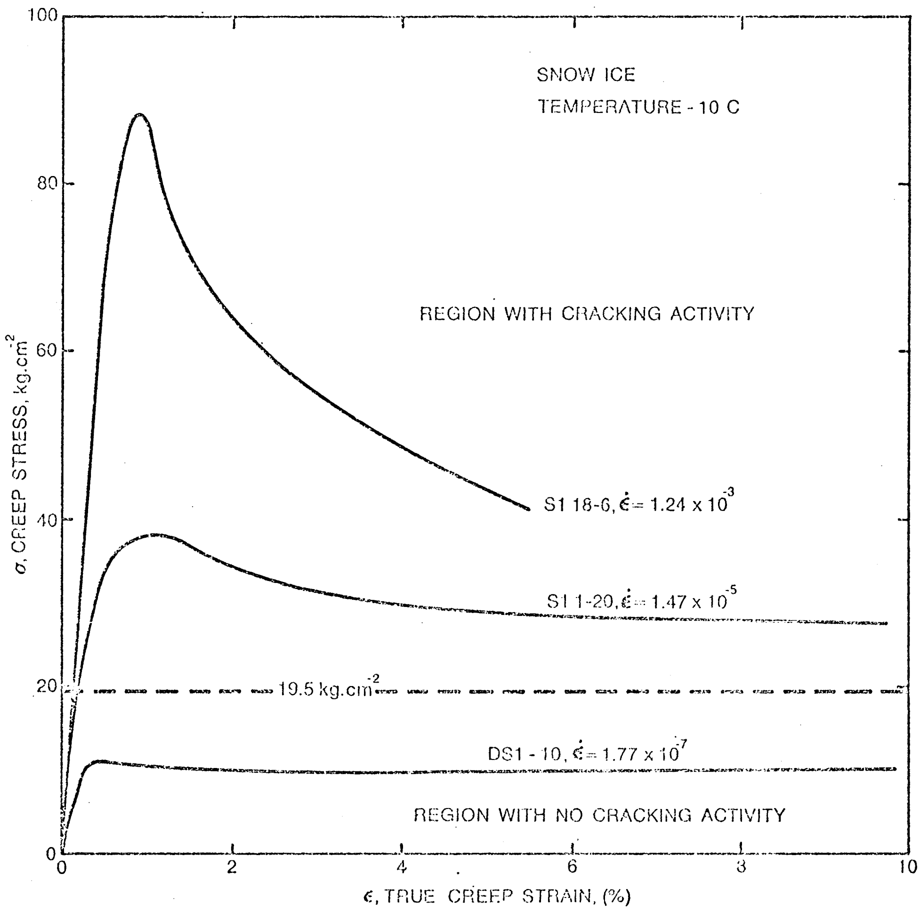

Figure 3 Compression creep curves conditions obtained under constant strain rate.

the theory, the data points have been plotted in figure 5 with the curves indicating the predicted values based on the theory. The agreement is very good. The dashed lines indicate the transition from one zone to the next for the lower line and from ductile to quasi brittle behavior for the top line.

APPARENT ELASTIC MODULUS FOR SNOW ICE

Each point in figure 6 represents an average of 4 tests at the same temperature. A curve has been fitted by least square analysis to the data of snow ice which is represented below in terms of an equation in kg.cm^{-2} where θ is given in degrees centigrade.

$$E_{TI} = \left(524.23 - 6.73\,\theta\right) 10^2 \tag{12}$$

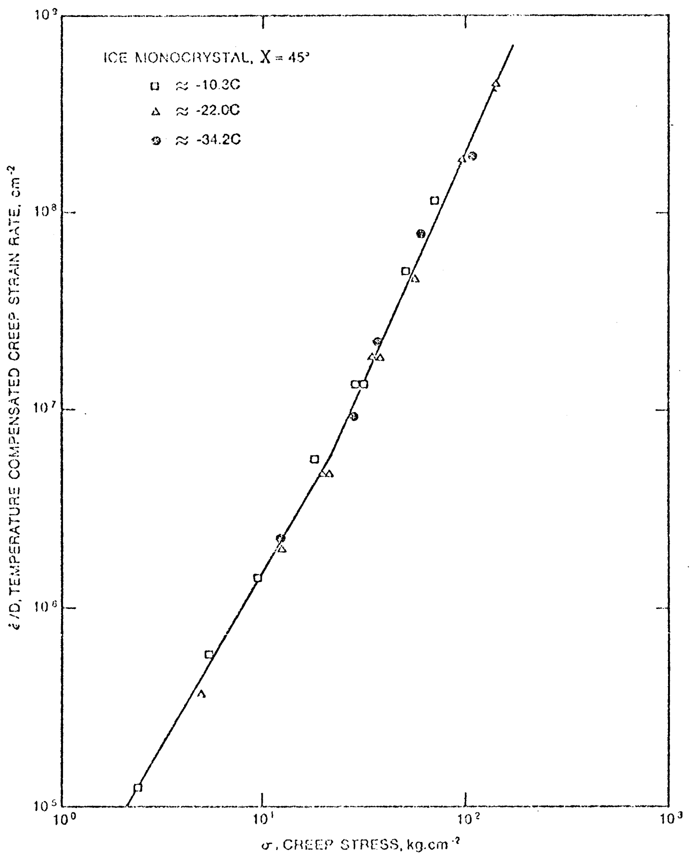

Figure 4 Temperature compensated $\dot{\epsilon}$/D versus σ for naturally grown ice monocrystals.

LABORATORY SNOW ICE RESULTS

All the results of the compression and tension tests have been combined in figure 7 using the relationship for the temperature effects. This results in a graph of $\dot{\epsilon}$/D versus σ/E. The fit of the data is remarkable. The results are grouped into two parts.

$$\dot{\epsilon}/D < 10^5 \qquad K = 4.712 \times 10^{15} \qquad n = 3.118 \qquad (13)$$

$$\dot{\epsilon}/D > 10^5 \qquad K = 1.297 \times 10^{23} \qquad n = 5.27 \qquad (14)$$

ANALYSIS OF RESULTS

As illustrated in figure 7, the intermediate stress range is not represented by a single power law but by two regions with different exponents. The low stress - low strain rate range has a smaller exponent than the high stress - high strain rate range. It is interesting to note that this difference is due to the appearance of cracks in the region with the higher exponent n. Therefore, the

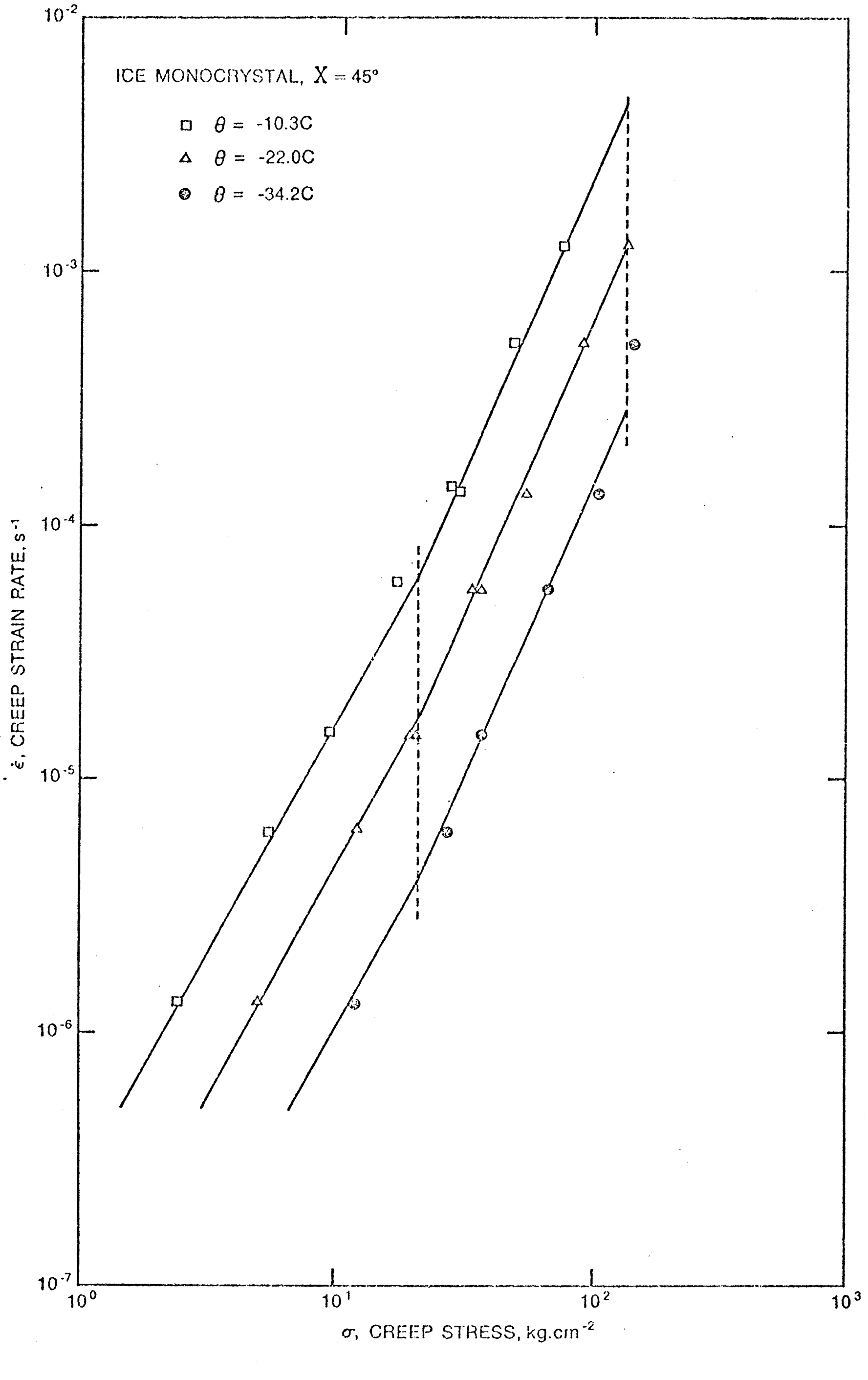

Figure 5 Strain rate versus stress as a function of temperature. Points indicate test results and lines represent computed curves based on figure 4.

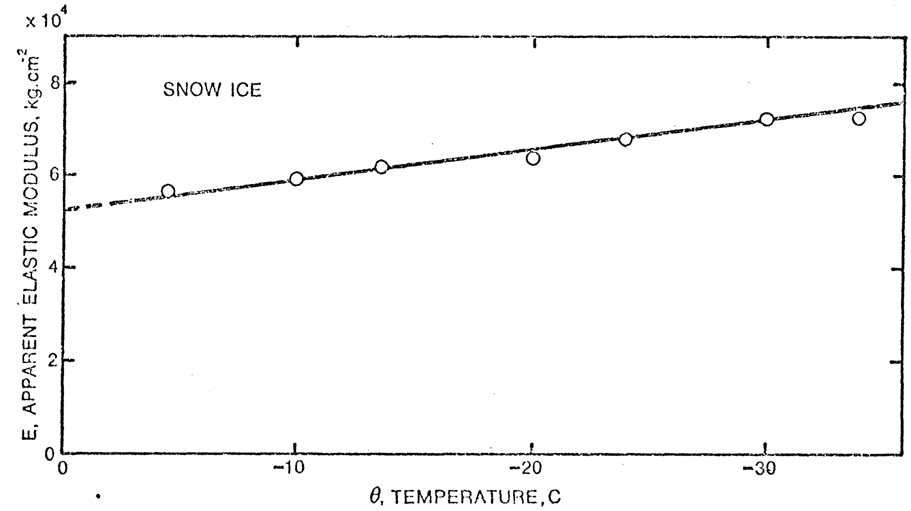

Figure 6 Apparent elastic modulus as a function of temperature. Each point represents an average of four tests at a given temperature.

region around $\dot{\epsilon}/D=10^5$ can be considered to be within the ductile range as illustrated in figure 8. Four regions have been designated based on different operating mechanisms. The general trend in the ductile-brittle transition zone and brittle behavior domain was obtained by Carter[8]. The limits of the stage II creep extend from near 10^1 to 10^9, similar to the behavior of metals[2]. Stage III already shows the appearance of brittleness, a deviation from the high stress range in metals where ductility is still maintained.

COMPARISON WITH OTHER RESULTS AND DISCUSSION

The results of the various authors discussed including those of the present study have been redrawn in figure 9. The agreement between Steinemann[9,10] and the present study is indeed very good. The results by Mellor et al.[11,12] fall within the same area as Glen's[13,14] results. The slopes are approximately the same. The difference between the four groups in the low strain rate region ($\dot{\epsilon}/D < 10^5$) is the angularity of the grains. There is no doubt that Glen's ice was very angular since the refrozen fresh frost particles did not undergo significant metamorphism. The snow used in the present study, however, was stored at approximately -12C for

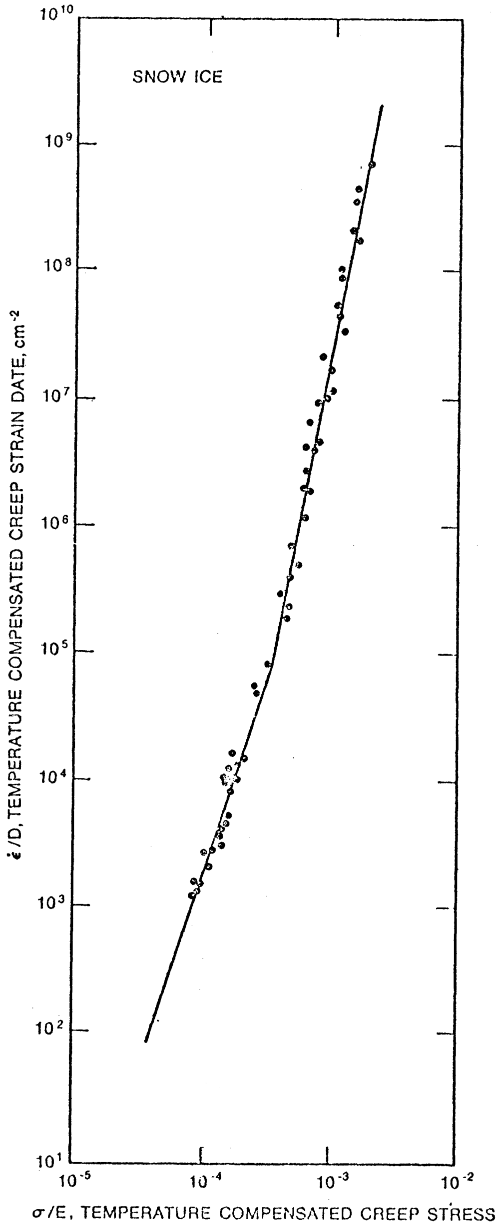

Figure 7 Diffusion compensated creep rate $\dot{\epsilon}/D$ as a function of σ/E for laboratory ice for constant strain rate and stress.

10 months. During this period sintering [15,16] took place rounding off grains making them more and more equiaxed. Mellor et al. used snow which probably was more angular either because of its age or state at the time of collection. At present the only variable which could explain the differences is the apparent elastic modulus. In all five cases the same modulus has been used as for the present

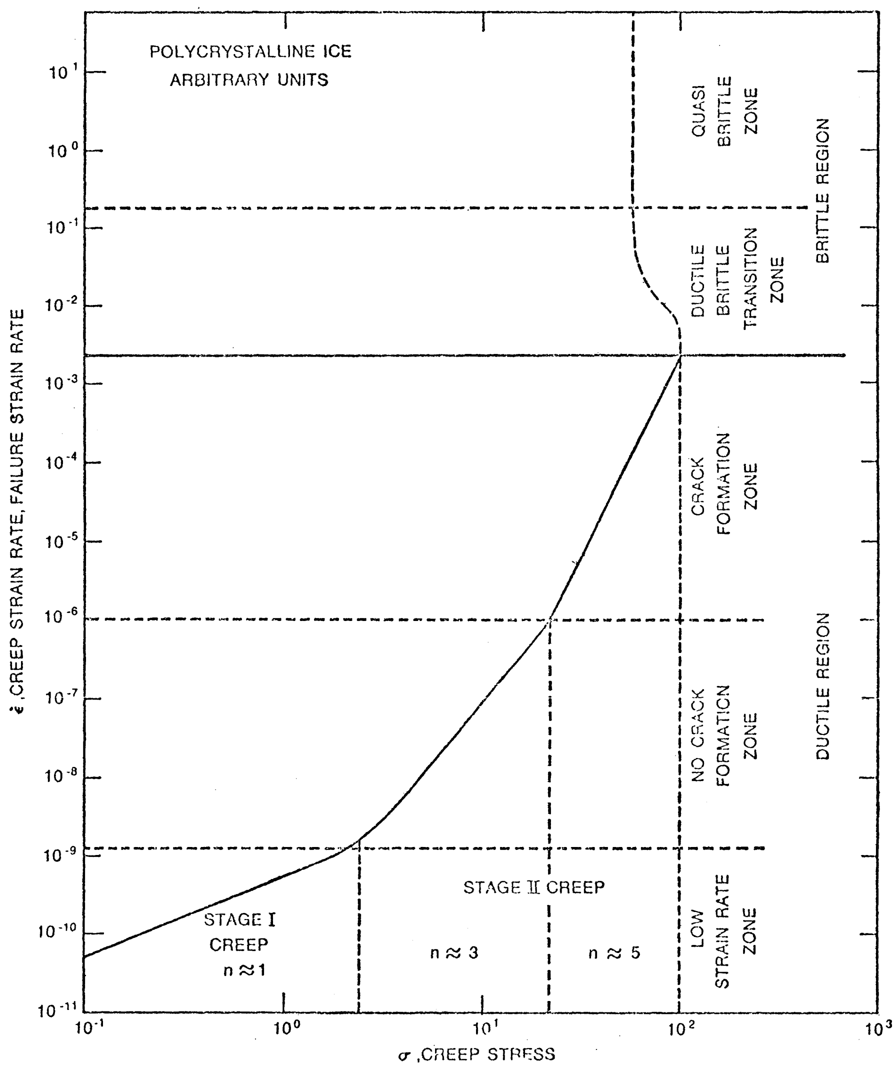

Figure 8 Graph illustrating the different zones of the strain rate-stress curve based on the material behavior.

analysis. In considering Glen's data it can be observed that increasing the apparent elastic modulus by a factor of 1.46 would give agreement with Steinemann's and the present results. At -10C this would mean an increase in the elastic modulus from 5.32×10^4 to 7.75×10^4 kg.cm^{-2} which is very much in the realm of possibility.

For engineering purposes it is very convenient to represent the data in the form of equation 8 to determine the creep strain rate or the form of equation 15 to evaluate the flow stress:

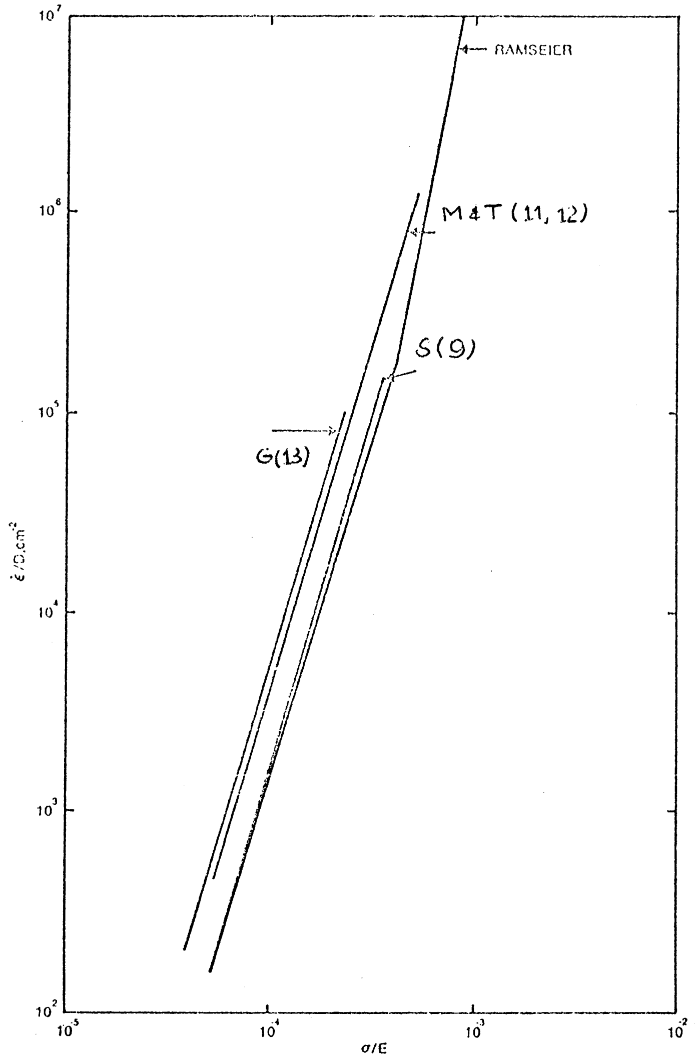

Figure 9 Comparison of creep behavior of data from various investigators including the present. In the case of the other data, the elastic modulus determined for the laboratory snow ice was used. By making a slight correction in the magnitude of the elastic modulus the curves by Glen[13] and Mellor and Testa[11,12] can be brought to coincide with the one by Steinemann[9] and the present study.

$$\sigma = \left[E \frac{\dot{\epsilon}}{KD}\right]^{1/n} \qquad (15)$$

Figure 10 represents the calculated curves obtained from the data in figure 7. Here the strain rate versus stress has been plotted as a function of temperature.

Data from Butkovich and Landauer[17] have been added to represent stage I creep, i.e., in the low stress region. The dash line which separates the low stress from the intermediate stress creep should be taken with some caution. It is very likely that the low stress transition could be shifted downwards. Field tests are required to determine this lower limit. The broken line in the intermediate range indicates the strong dependence of the transition zone on the strain rate separating the region of no crack formation from the one of crack formation. The top broken line indicates the transition from the ductile to the ductile brittle zone.

The stress dependence of the temperature compensated strain rate has been plotted against temperature on figure 11. The three transition curves are shown starting with the low stress-intermediate stress transition at $\dot{\epsilon}/D = 1.52 \times 10^2 cm^{-2}$ at the bottom followed by the no cracking activity - cracking activity transition at $\dot{\epsilon}/D = 8.4 \times 10^4 cm^{-2}$ and the ductile - ductile brittle transition at $\dot{\epsilon}/D = 3.18 \times 10^8 cm^{-2}$. In addition, the curve for $\dot{\epsilon}/D = 10^7$ in the cracking activity zone has been added. As can be seen the temperature dependence increases with an increase in $\dot{\epsilon}/D$ values.

Figure 12 shows the temperature effect at a constant strain rate. The bottom curve at a $\dot{\epsilon} = 2 \times 10^{-9}$ shows the behavior in the no crack zone followed by a curve at $\dot{\epsilon} = 10^{-7}$ which passes through the crack transition slightly below -30C. The $\dot{\epsilon} = 3 \times 10^{-6}$ shows the curve in the crack zone and for $\dot{\epsilon} = 2 \times 10^{-4}$ the curve again passes through a transition, the ductile - ductile brittle one at a temperature of -35C. At -50C this curve merges with the quasi-brittle curve at a $\dot{\epsilon} = 10^{-1}$ which exhibits a linear behavior since

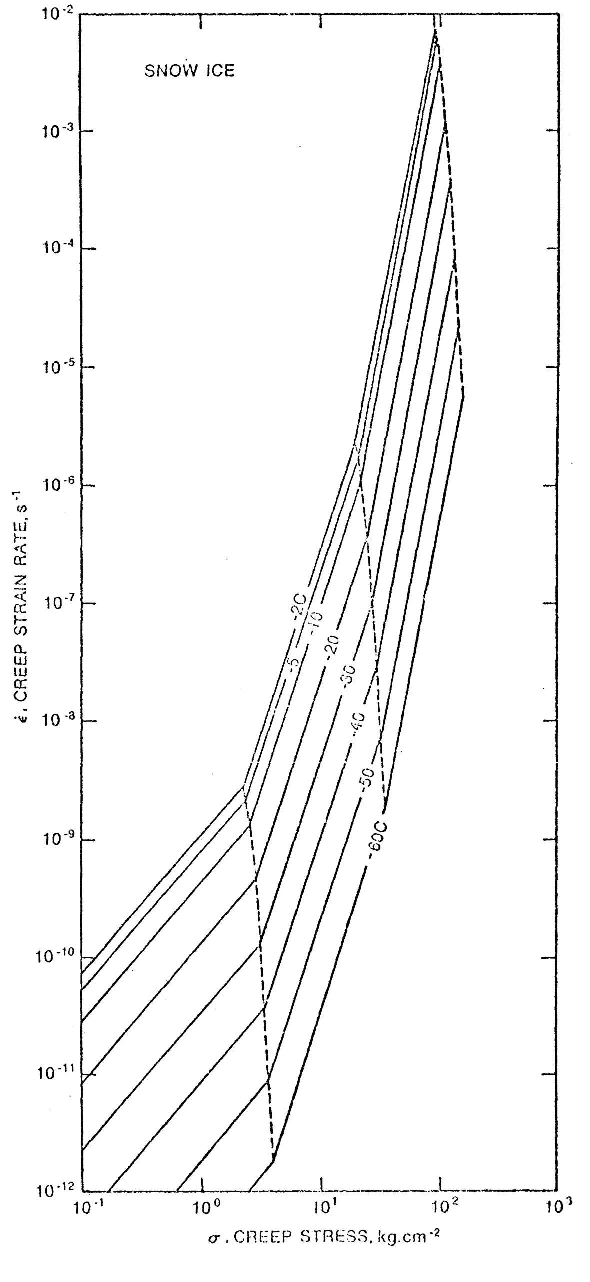

Figure 10 Creep strain rate versus stress as a function of temperature for the intermediate and low stress range of snow ice.

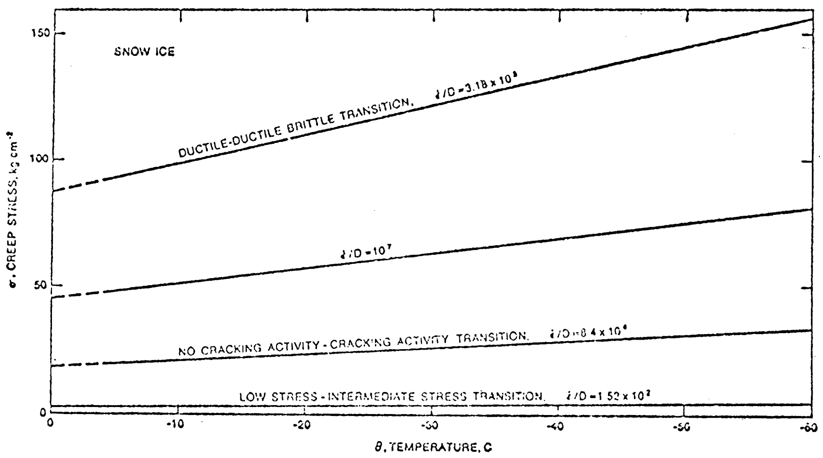

Figure 11 Creep stress versus temperature as a function of the temperature compensated $\dot{\epsilon}/D$. Three transition regions are indicated based on those shown in figure 10.

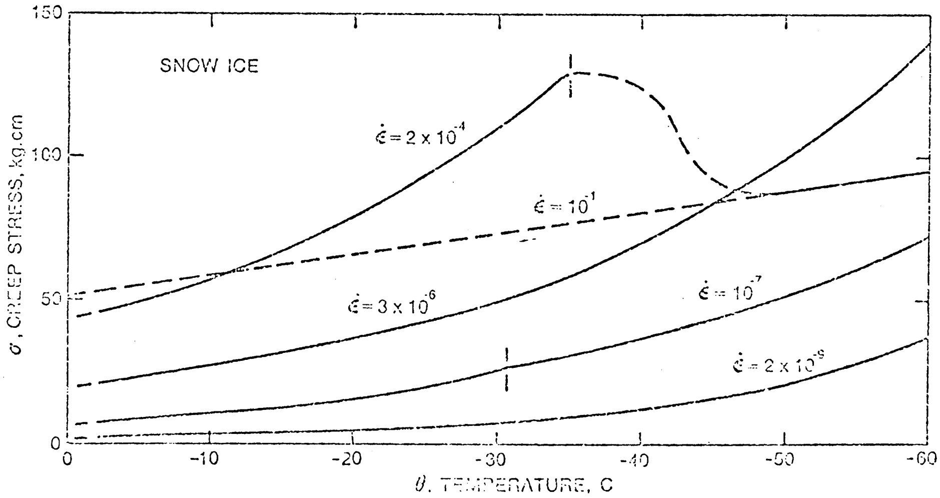

Figure 12 Creep stress versus temperature as a function of strain rate. Some of the curves cut across transition as indicated by short vertical broken lines.

the strain rate is independent of the stress in this region[8].

SUMMARY

The present study of snow ice in the laboratory to determine its general behavior under constant stress and strain rates has led to a general power law relationship which brakes down into two parts in the ductile region. The higher exponent $n \approx 5$ describes the region which contains cracks where as $n \approx 3$ describes the region containing no cracks.

There is excellent agreement with other published data if they are treated in the same way as those of this study. By making slight corrections in the apparent elastic modulus, grain size dependence, and grain shape effect, complete agreement could be obtained.

The work reported here is part of a general study which includes the behavior of frozen frazil slush, columnar ice, the effect of density and impurities, and the effect of meteorological and hydrodynamic parameters on the formation of the various ice types.[7]

* The experimental work was done while on educational leave at Laval University.

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