PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
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FORCES IN MOVING ICE FIELDS

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Abstract: Forces which moving ice fields exert on structures depend upon size and shape of the structure ranging from isolated piles to straight walls and sloping cones. Broken-up ice causes additional design difficulties. Ice properties and failure modes must be considered.

If one freezes an isolated pile into an ice field and the ice starts to move tremendous forces can develop but not more than the ice itself can stand. A nominal crushing strength

$$\sigma_n = \frac{P}{hB} \tag{1}$$

can be calculated with P - force developed on the pile, h - ice thickness, B - diameter of pile or width of structure.

Generalizing of such results for larger structures leads to overdesign and could delay progress for a generation. One must consider the way the ice fails around larger structures when ice fields move against a random collar of broken-up ice, close to impossible to define. The purpose of this paper is to outline conceptual difficulties and to prepare a transition for solution.

One could make the argument that ice could adfreeze smoothly to a structure without a collar of randomly oriented pieces. This may be true in a lake but not in the ocean with its continuous tides. Structures, as well as islands are surrounded by collars of randomly oriented blocks. Therefore, design against ice forces for maritime structures is quite different from design for structures in lakes.

Current engineering thought is concentrated on sloping structures, such as conical surfaces to facilitate breaking of ice. Many engineers are tempted to use equations of classical plate mechanics and will dangerously underdesign structures. Ice does not behave in such a simplistic manner. Rather than to break smoothly it piles up on the sloping structure. One could use principles of soil mechanics to calculate forces during such a pile-up but nobody knows the required parameters, such as internal friction and cohesion of loosely piled up ice masses. Some people feel tempted to solve the problem by using model tests with artificial materials. This gives valuable insight but ice in nature can freeze forming a solid mass of piled-up pieces. There are many more problems in addition to those stated.

Take equation (1) and apply it for a straight wall

$$\frac{P}{B} = c\,\sigma_\infty\, h \tag{2}$$

with σ_∞ - the crushing strength for a straight wall which is clearly different from σ_0 - the nominal crushing strength for a pile approaching $B/h = 0$. c - is the contact coefficient as suggested by Korshavin (1962). In case if the edge of a semi-infinite ice sheet approaches the pile identation occurs. Depending upon the theory one uses different ratios $S = \frac{\sigma_0}{\sigma_\infty}$ can be computed but it usually amounts to somewhat less than 3.

In addition a shape factor (semi-circular, triangular or straight-edge indenter) is involved which we shall not consider here.

One could postulate

$$S = \frac{\sigma_n}{\sigma_\infty} = 1 + \left(\frac{\sigma_0}{\sigma_\infty} - 1\right) 2^{-\beta/\beta_0} \tag{3}$$

with $\beta = \frac{B}{h}$ and β_0 the value where $2^{-\beta/\beta_0}$ becomes $\frac{1}{2}$ in an exponential approach from $\sigma_n = \sigma_0$ for $\beta_0 = 0$ and $\sigma_n = \sigma_\infty$ for $\beta = \infty$ as defined.

The result is a curve in which the all important σ_∞ is difficult to determine. The exponential e^{-x} is linear in good approximation for small x. We do not know for what value B/h S becomes $\frac{1}{2}$ but sofar $\beta_0 = B/h = 1$ has given good results. By plotting S versus h/B (the reciprocal) rather than B/h which tends to infinity we can linearize S for $\beta/\beta_0 > 1$ and the straight wall $B/h = \infty$ comes within plotting range for $h/B = 0$. Then by plotting S versus β/β_0 for β/β_0 between 0 and 0 and versus β_0/β between 1 and 0 we achieve a curve which within measurement accuracy can be assumed as linear. The intercept $\beta_0/\beta = 0$ gives σ_∞ for a straight wall and the intercept $\beta/\beta_0 = 0$ σ_0 for a thin pile being attacked by the edge of an ice sheet.

Here comes the next problem. σ_0 / σ_∞ may be slightly below 3 at the edge but could be much higher in the interior of the ice field for a case such as discussed in the beginning. As a matter of fact theoretically it could be infinite for $h/B = \infty$ simply because the ice when failed has nowhere to go. Only regelation, resembling the flow in a highly viscous liquid such as ice could bring the value down from infinity. In practice, however, the ice will move up and down perpendicular to the available free surfaces in a poorly understood and little studied manner.

One can raise the question whether σ_∞ can be related to the crushing strength σ_c of a small cylindrical sample. The geometry of the failure surface (cone in the laboratory, oblique plane in nature) does not affect the result directly. This seems to hold for an isotropic medium, but sea ice is not isotropic being composed of vertical columns parallel to the growth direction. Laboratory tests (Weeks and Assur 1969) have shown that σ_c can easily exceed 1000 to/m^2 if the force is applied parallel to the growth direction. In case of a straight wall the compressive force is applied perpendicular to growth but due to geometry the slip planes form diagonally across the columns as in the laboratory with a force applied parallel to growth.

The crushing strength in the laboratory perpendicular to growth is much less. Presumably one could use this value for an edge load with Prandtl type indentation, negating thereby the effect of equ (3) since σ_0 is considerably less than in case of anisotropic sea ice. The Prandtl type failure, however, does not occur as soon as the pile penetrates into the ice sheet. The ruptured material moves up and down rather than sideways.

Another difficulty in relating laboratory tests to field conditions is the size of the sample. It does matter as shown in Weeks and Assur (1969). By plotting $\frac{1}{\sigma}$ versus $\sqrt{F} = \sqrt{Bh}$ one obtains the linear portion of a general exponential relationship, the asymptote of which for $F \to \infty$ is not known as yet. Laboratory tests will give exaggerated values for field applications.

The size of the ice pieces or ice fields attacking a structure has an effect on forces developed since the mass and therefore the kinetic forces simply may not be sufficient to engage the full crushing force of ice. Usually this size effect is not of consideration since sufficiently large ice fields can be found. Exceptions are narrow rivers during ice breakup or the southern portion of the Caspian Sea where local ice does not develop but drifting broken up ice fields may endanger offshore structures.

σ_∞ for fresh ice has a slight dependence upon temperature as long as the ice surface is covered with snow or snow ice. However, without snow fresh ice upon rapid decrease of air temperature is liable to excessive thermal cracking and considerable loss of bearing capacity. Such temperature changes could also affect forces on structures. Once ice starts to melt in the spring its strength rapidly deteriorates depending upon absorbed radiation which can be calculated from heat balance. This, in turn, can be related to the sum of positive air temperatures, $\Sigma\,\Theta_a$. In addition to the loss in strength the ice thickness diminishes also proportional to $\Sigma\,\Theta_a$. As a result traffic over ice becomes extremely hazardous. For river structures which are being attacked by ice after breakup considerably weakened ice can be assumed. For maritime structures, however, full winter conditions must be assumed.

For sea ice all strength properties including σ_∞ are strong functions of brine volume which in turn can be calculated from temperature and salinity (Assur, 1958). As a general rule

$$\frac{\sigma_s}{\sigma_r} = \left(1 - \sqrt{S_i \nu_1}\right)^2 \tag{4}$$

σ_s - strength of sea ice, σ_r - strength of fresh ice,

$S_i \nu_1$ - brine volume

S_i - salinity of ice, ν_1 - a function of temperature

In practice σ_∞ can be calculated using temperature and salinity for top and bottom and two equidistant points in between, calculating the brine volume, expressing the resulting profile in a series and integrating the series as suggested by Assur (1967) and also shown in Weeks and Assur (1968).

The constant C in equ (2) accounts for the fact that crushing cannot occur along the entire wall simultaneously. Once a piece of ice breaks, the resistance at this point decreases substantially while other locations are subjected to the full crushing force. Obviously for a small area full crushing strength can be mobilized, C depends therefore upon size. Statistically C could be 1/2 but soft, pliable ice may utilize the full crushing strength along the entire w all with $C \to 1$, for highly brittle ice C will be appreciably less. This leads to another paradox. Cold ice has high strength but it is brittle and produces a small C , warm ice has low strength but is highly pliable with a high C . As a result soft weak ice could exert a higher force than cold ice. Some field data seem to confirm this.

The relationship (3) in a linearized form can be easily checked in a model test deriving σ_∞ (for a wall) and σ_0 (for an edge), as the case may be, as intercepts. However, in model tests for large B/h one runs into stability failures rather than crushing failures. Buckling must also be considered in areas with relatively thin ice such as Norwegian waters, the Baltic Sea or Cook Inlet in Alaska.

Buckling instability against a straight wall is governed by the equation

$$\frac{P}{B} = 2 K \ell_1^2 \tag{5}$$

$\frac{P}{B}$ - load for unit width

K - foundation modulus or density of water

$\ell_1 = \sqrt[4]{\dfrac{E h^3}{3 K (1-\mu^2)}}$ action width

E, μ - Young's modulus and Poisson's ratio of ice

> Note: Eq (5) holds for an infinite beam, a hinged and a fixed semiinfinite beam. A semiinfinite beam with a free end gives half of the value of equ (5) but in nature the end is more or less restrained so that a coefficient approaching 2 would be more appropriate than 1.

In sea ice E is a strong function of depth within ice because of temperature and salinity. The general relationship is

$$\sqrt{\frac{E_s}{E_r}} = (1 - S_i \nu_1)^2 \tag{6}$$

with E_s, E_r - Young's modulus for sea ice and fresh ice, respectively.

Classical theories deal with a homogeneous material (uniform E). Assur (1967) has suggested how to cope with a variable Young's modulus. Weeks and Assur (1968) show relationships for practical use. Note, however, the corrected equation (6) given here.

To test the relationships given here it is convenient to form the ratio

$$R = \frac{P}{B h \sigma_c} \tag{7}$$

One may also use the flexural strength σ_f if measured. σ_f can be assumed proportional to σ_c . Another possibility is to use equation (4) integrated over depth as a function of measured temperature and salinity.

Plotting R versus $1/\beta = h/B$ and versus B/h for $\beta/\beta_0 \leq 1$ one should obtain the intercept for buckling when thin ice attacks an infinite wall $h/B \Rightarrow 0$. For this purpose equ (5) must be expressed in compatible parameters.

Since $\ell_1 = L_1 h^{3/4}$ with $L_1 = \sqrt[4]{\frac{E}{3k(1-\mu^2)}}$ and E can be expressed as $E = a\sigma_c$ we have

$$\frac{P}{hB\sigma_c} = 2\sqrt{\frac{a}{3(1-\mu^2)}\,\frac{kh}{\sigma_c}} \tag{8}$$

Finally let us estimate forces if confronted with a collar of randomly oriented ice pieces. These can form around structures, islands or as pressure ridges between drifting ice fields.

At this time let us confine ourselves to a two dimensional model. If the edge of an ice sheet strikes a given piece of ice this piece can be oriented at an aribitrary angle θ to the horizontal.

Defining $tg\,\phi$ as the friction coefficient the ratio of horizontal (H) to vertical force (V) against the sloping surface is

$$\frac{H}{V} = \frac{\cos\theta - \sin\theta\, tg\,\phi}{\sin\theta + \cos\theta\, tg\,\phi} = tg\,(\theta + \phi) \tag{9}$$

The ice sheet will fail under bending (slab on a liquid foundation) up to a certain angle θ_1, when crushing will occur (or buckling for thinner ice). One can define a "probability ellipse" which represents the likelihood of a certain angle θ to occur. Horizontal ice pieces may be preferred in case of loose pressure, vertical ice pieces in case of intense pressure. In addition one must consider the possibility of "wedging" after initial failure occurred. Let us assign equal probabilities to all angles. In that case the integral

$$tg\,(\theta + \phi)\, d\,(\theta + \phi) = -\frac{2\theta_1}{\pi}\, ln\cos\,(\theta_1 + \phi) \tag{10}$$

leads us to a statistical average of rather undefinable forces. θ_1 itself is the angle of an ice piece under which failure will not occur in bending but in crushing. This angle can be cal culated from the condition

$$F\, tg\,(\theta_1 + \phi) = c\,\sigma_c\, h \tag{11}$$

with

$$F = \frac{\sigma_f h^2}{6\ell_1}\sqrt{2}\, e^{\pi/4} \tag{12}$$

where F represents the vertical force (per unit width) required to break a semi-infinite beam on a liquid foundation.

From (11) and (12)

$$tg(\theta_1 + \phi) = \frac{6c}{\sqrt{2}\, e^{\pi/4}} \cdot \frac{\sigma_c}{\sigma_f} \frac{\ell_1}{h}, \qquad (13)$$

with $\frac{\sqrt{2}\, e^{\pi/4}}{6}$ = 0.51696, depends largely upon the ratio of crushing strength to flexural strength.

This paper, by necessity, is limited to a conceptual outline of numerous problems the engineer finds himself confronted with attempting to design structures subjected to moving ice fields. In particular a suggestion is made how to deal statistically with randomly broken up ice collars. Further details remain to be published in subsequent papers.

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