ABSTRACT
Fracture of sea ice plays an important role in determining the ice loads on offshore structures. However, any method developed based on fracture mechanics demands the knowledge of this rather important parameter, i.e., the fracture toughness of sea ice. There have been controversies about the scale invariant fracture toughness $K_I$ of sea ice for over 25 years now. E.g., $K_{IC} = 115 \text{kPa} \sqrt{\text{m}}$ was reported from laboratory measurement while $K_{IC} = 250 \text{kPa} \sqrt{\text{m}}$ was reported from field scale. The size of the test specimen and the loading rate are the two major issues often raised when questioning the validity of a fracture toughness test. In view of such discrepancy in the reported values of fracture toughness, the Norwegian University of Science and Technology (NTNU) hosting the research-based innovation centre (SAMCoT), is making an effort to attack this long-existing problem. Two field tests in Svea at Spitsbergen are planned in the winters of 2015 and 2016. The rationale of the test is rather straightforward, i.e., we shall fracture a large enough ice floe with fast enough loading rate. The test in 2015 is a preparatory campaign for the main testing campaign in 2016. Based on existing fracture mechanics knowledge and precursors’ experience with fracture toughness test, this paper describes the design process of the first test campaign with emphasis on the theoretical analysis. Interesting results regarding the size and loading rate requirement are obtained. By this paper, we hope to expose our work at this early stage to discussions, suggestions and criticism from worldwide experts in the field. Based on this test design, the first test campaign shall be carried out between March 20th-27th, 2015, after which we report the test results in a different paper separately.
1. INTRODUCTION

Ice fracturing often occurs while a large ice floe interacts with various types of offshore structures. Different failure modes (i.e., bending, buckling, crushing, splitting, etc.) documented in various engineering codes (e.g., (API_RP2, 1995; ISO/FDIS/19906, 2010)) are essentially various fracturing mechanisms (Lu, 2014; Lu et al., 2015d). However, while dealing with these failure modes, people usually adopt the strength theory based approach. This might be feasible for local failures which are controlled by fracture initiation (Lu et al., 2015b), whereas large cracks often observed in the field demand knowledge of fracture mechanics. Palmer et al. (1983) highlighted the importance of fracturing as a limiting mechanism in determining the ice force. Similarly, many previous researchers expressed the wish of including fracture mechanics into a probabilistic load model to be less conservative (e.g., (Kennedy et al., 1993)) in ice load calculations. However, before we can achieve these goals, one fundamental question remains to be resolved, i.e., what is the fracture toughness of sea ice?

Numerous excellent work has been conducted to measure the so-called “fracture toughness” of ice (see review papers such as by Dempsey (1991) and by Schulson and Duval (2009)). Currently, there are two typical numbers in the ice research community for the fracture toughness of sea ice (Timco and Weeks, 2010), i.e., one is 115 kPa√m from laboratory tests, and the other one is 250 kPa√m based on tests on large test specimens.

Most laboratory tests employed the methodologies of Linear Elastic Fracture Mechanics (LEFM). Schulson and Duval (2009) summarised different test methods and important test conditions. One important condition for the validity of LEFM is the so-called Small Scale Yield (SSY) condition (Rice, 1965). That demands a small inelastic zone (for ice, the creeping zone) comparing to the size of the tested samples. It has been a long research journey in looking for the proper size requirement for the validity of LEFM in ice fracture toughness test. The typical plastic zone estimation formula (e.g., see (Anderson, 2005)) is not applicable for sea ice due to its large grain size and high homologous temperatures (Dempsey, 1991). In an excellent review paper, Dempsey (1991) pointed out the following approach to take in order to obtain the true ice fracture toughness as a material property:

- Test on large size samples such that notch sensitivity can be ensured;
- Adoption of nonlinear fracture mechanics.

Thereafter, six field trips were made by these authors to test the ice fracture toughness in large scale (Adamson and Dempsey, 1998; Adamson et al., 1995; Dempsey et al., 1999a; Dempsey et al., 1999h). After a series of data analysis and theoretical development (i.e., fictitious viscoelastic fracture model), a fracture energy $G_T = 15 \text{ N/m}$ (e.g., corresponds to the fracture toughness of 250 kPa√m (Dempsey et al., 1999a)) and a Traction-Separation Law (TSL) curve were proposed (Mulmule and Dempsey, 1997; Mulmule and Dempsey, 1998; Mulmule and Dempsey, 1999). These results are twice different from those ‘subsized’ laboratory test results.

However, Schulson and Duval (2009) pointed out that the loading rates in the field tests conducted by Dempsey’s team were too slow to impede creep effect, thereby raising the fracture toughness value. Though the viscoelastic material properties were taken into account within the bulk test specimen, a rate dependent TSL was not included in the model of Mulmule and Dempsey (1997; 1998; 1999).
Driven by the importance of sea ice’s fracture toughness, and based on all the precursors’ experience, the aim of this paper is to design a field test that a large enough specimen can be loaded fast enough to evaluate the fracture properties of sea ice.

2. OVERVIEW OF THE TEST DESIGN

The field test site is chosen at the bay Braganzavågen in the Van Mijen Fjord of Spitsbergen (Figure 1a, b and c). The fjord is about 50 km long and spreads into Spitsbergen from the West to the East at about 77.8°N (Figure 1b). At the mouth, the fjord has width about 10 km, while to the head of the fjord it reduces down to 5 km. There is the Akseløya island, at the mouth of the fjord, which blocks the fjord almost completely. This facilitates the process of stable land-fast ice formation. Water depth at the chosen test location is in a range of 5 to 10 metres. Usually, at the head of the fjord, stable ice conditions prevail from January until June and thickness of sea ice can be expected to reach value about 1 m, depending on the season. There is a coal mining settlement, Svea, located at the head of the fjord (Figure 1b). It is relatively easy to access with snowmobiles during the winter season. All this: stable ice conditions, easy transportation access and established infrastructure in Svea; made Van Mijen Fjord a favourite research site for this field test campaign.

There are in total two test campaigns under plan. One preparatory test is scheduled from March 20th to 27th, 2015. Based on the experience accumulated in this preparatory test, a more comprehensive test shall be conducted in the same place in the winter of 2016. The aim of the first test campaign are as follows:

- To test the initially prepared equipment and data logger system;
- To familiarise ourselves with all the important equipment and data logger system;
- A simplified one parameter fracture toughness \( K_I \) test is carried out following the principle of LEFM;
- Experiences gathered in the first test campaign are transferred to a next phase test to extract the multi-parameters controlled fracture properties.
The methodology behind the first test campaign is rather straightforward, i.e., we shall test on a large enough specimen with fast enough loading rate. Assuming the applicability of LEFM under these two conditions, the loading history and displacement history are measured to extract the expected $K_c$ value for sea ice. Before the actual test, this paper aims at answering these two questions (i.e., how large and how fast) quantitatively based on existing fracture mechanics theories.

In the following, detailed information regarding the test set-up and preliminary theoretical studies are conducted in relation to the design of the experiment.

3. GEOMETRY AND SIZE CONSIDERATIONS

3.1 Geometry of the test sample

Different geometries of the test specimen have been tested in the field (Adamson et al., 1995; Dempsey et al., 1999a; Dempsey et al., 1999h). Based on these authors’ experience (e.g., ease in preparation), we adopted a similar test geometry in the current field test, i.e., an Edge Cracked Rectangular Plate (ECRP, see Figure 2). In addition, theoretical development of the weight function for ECRP with different width to length ratio ($W/L$) has been completed in a recent paper (Dempsey and Mu, 2014); and adopting a similar test geometry allows us to cross check with previous tests’ analysis and results.

Dempsey and Mu (2014) proposed weight functions for ECRP with 6 width to length ratios, i.e., $W/L = 0.25, 0.5, 1, 1.5, 2, 4$, while it was mainly the square ice plate (i.e., $W/L = 1$) that were tested and analysed in previous field experiments (Dempsey et al., 1999a; Dempsey et al., 1999h; Mulmule and Dempsey, 1998; Mulmule and Dempsey, 1999; Mulmule and Dempsey, 2000).

Among these choices, we try to avoid long plate (Figure 2a) and choose wide plate (Figure 2b). This is because we try to make the potential crack propagate in a self-similar manner (as shown in Figure 2b). From the plastic limit theory point of view (Lu et al., 2015a), the crack tends to travel the shortest distance so as to minimize the overall energy dissipation. Choosing a long ice plate, we are running a higher risk that the crack would kink sideways by either inhomogeneity or an imbalanced loading.

However, a wider test sample means more test preparation. In the current test design, we choose a wide plate with $W/L = 2$ as the basic geometric configuration.

![Figure 2. Edge Cracked Rectangular Plate (ECRP) with different width to length ratios.](image-url)
3.2 Size requirement to ensure notch sensitivity

After defining the basic geometry, we still need to quantify the size of the test sample. How large should the ice plate be to be large enough? Following the relationship between test specimen size and notch sensitivity originally proposed by Carpinteri (1982) in concrete’s fracture tests, Dempsey (1991) proposed the notch sensitivity criterion (see below) for sea ice tests. Notch sensitivity is defined in Eq. (1). It represents the ratio between the critical nominal tensile stress $\sigma_N$ at the crack tip and the flexural strength $\sigma_f$. This ratio is a function of specimen size defined by the brittleness number $\beta_Q$, which is indicative for the applicability of LEFM in a fracture test (Mulmule and Dempsey, 2000).

$$\frac{\sigma_N}{\sigma_f} = C_N \beta_Q$$  \hspace{1cm} (1)

in which, $C_N$ is a constant parameter for a given crack geometry. In a preliminary estimation of the size requirement, we consider an ECRP loaded by a concentrated force $P$ at the crack mouth. The nominal strength $\sigma_N$ of this cracked configuration is given in Eq. (2).

$$\sigma_N = \frac{2P(A+2L)}{t(L-A)^2}$$  \hspace{1cm} (2)

While at crack initiation, the critical concentrated force $P$ is given in Eq. (3). (originally from equation (20) of Dempsey and Mu (2014))

$$\frac{P}{tK_Q\sqrt{\pi A}} = (1-a)^{3/2} \frac{F_p(a)}{F_p(a)}$$  \hspace{1cm} (3)

in which,

- $P$ is the critical force just at the beginning of crack propagation and it acts at the crack mouth [kN];
- $t$ is the thickness of the test specimen [m];
- $K_Q$ is the apparent fracture toughness of the test specimen (not knowing beforehand), but two typical values $K_{Q,e}$ and $K_{Q,L}$ are utilised in back calculations to estimate the size and loading rate requirement [kPa m$^{-1}$];
- $A$ is the size of the crack, e.g., see in Figure 2, [m];
- $a$ is the normalised crack length with $a = A/L$, and $L$ is the specimen length as in Figure 2, [-];
- $F_p(s)$ is a function defined in Eq. (3), [-];
- $\alpha_i^p$ are parameters supplied in Dempsey and Mu (2014) for different width to length ratio, [-].

Inserting Eqs. (2) and (3) into Eq. (1), we have the following expressions for our selected geometry:

$$\frac{\sigma_N}{\sigma_f} = \frac{2\sqrt{\pi a(a+2)}}{F_p(a)\sqrt{1-a}} \frac{K_Q}{\sigma_f \sqrt{L}}$$

$$C_N = \frac{2\sqrt{\pi a(a+2)}}{F_p(a)\sqrt{1-a}}$$

$$\beta_Q = \frac{K_Q}{\sigma_f \sqrt{L}}$$  \hspace{1cm} (4)
The notch sensitivity is a function of crack length $a$ (imbedded in the parameter $C_N$) and specimen size $L$ (imbedded in the brittleness number $\beta_Q$). Recalling that a characteristic length $l_{ch}$ in the cohesive zone theory can be defined as in Eq.(5) (Hillerborg et al., 1976), the brittleness number $\beta_Q$ actually can be written as in Eq. (6) with a more straightforward physical meaning. Note that $l_{ch}$ is a material property and the cohesive zone at the crack tip of sea ice is approximately $(0.3 - 0.5) \cdot l_{ch}$, i.e., the physical size $L$ should be much larger than $l_{ch}$ to ensure the inelastic zone confined in a small region of the crack tip (Malmule and Dempsey, 2000). Hence the brittleness number $\beta_Q$ and notch sensitivity should be as small as possible. Based on a series of analysis, Dempsey (1991) proposed that $\sigma_N / \sigma_f$ is smaller than 0.5 to ensure notch sensitivity.

$$l_{ch} = \left( \frac{K_Q}{\sigma_f} \right)^2$$

$$\frac{1}{\beta_Q^2} = \frac{L}{l_{ch}}$$

Although the actual fracture toughness of sea ice is not known before the test, based on existing numbers, we assume $K_{QLT} = K_{QLT} = 250 \text{kPa} \sqrt{\text{m}}$ and $K_{QLT} = K_{QLT} = 115 \text{kPa} \sqrt{\text{m}}$ respectively so as to estimate the size requirement of the test specimen. In addition, we assume the flexural strength of sea ice $\sigma_f = 500 \text{kPa}$, and the notch sensitivity of our chosen test geometry with varying sizes are illustrated in Figure 3. The plot is comparable to the one for edge cracked square plate by Mulmule and Dempsey (2000).

Figure 3 demonstrates that in order to ensure notch sensitivity (i.e., $\sigma_N / \sigma_f < 0.5$), a larger test specimen is required if sea ice has a larger fracture toughness. The apparent fracture toughness $K_{QLT} = 115 \text{kPa} \sqrt{\text{m}}$ attained in the laboratory needs a specimen size approximately $L > 1 \text{m}$ to ensure notch sensitivity of the test. This indicates that most laboratory tests were subsized.

To ensure the required notch sensitivity, judging from the plots in Figure 3, it is generally required the test specimen size should be larger than 5 m. Moreover, a relative crack length $a = 0.3$ is preferred in the planned test for purpose of cross checking with previous field tests.

In order to cut free such a large ice floe, a Ditch Witch trencher (R20) is employed for this demanding task. An overview summary on the important parameters of this Ditch Witch trencher is illustrated in Figure 4 and Table 1 (DitchWitchOrganization, 2011). The machine weighs 508 kg with a potential trenching speed of 1 m/s. Based on Gold’s (1971) recommended formula, the minimum ice thickness to bear this machine is about 12 cm.
Figure 3. Notch sensitivity of an ECRP with width to length ratio of 2 under concentrated force at crack mouth.

Figure 4. Geometric specifications of the employed Ditch Witch trencher (R20) (DitchWitchOrganization, 2011).

Table 1. Geometric specifications of Ditch Witch trencher (R20).

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Trench depth, maximum</td>
<td>762 mm</td>
</tr>
<tr>
<td>B</td>
<td>Trench width</td>
<td>110-150 mm</td>
</tr>
<tr>
<td>C</td>
<td>Boom travel down</td>
<td>60°</td>
</tr>
<tr>
<td>C1</td>
<td>Boom travel up</td>
<td>60°</td>
</tr>
<tr>
<td>F</td>
<td>Head shaft height, digging chain</td>
<td>220 mm</td>
</tr>
<tr>
<td>L3</td>
<td>Length</td>
<td>2.1 m</td>
</tr>
<tr>
<td>W2</td>
<td>Width</td>
<td>840 mm</td>
</tr>
<tr>
<td>H2</td>
<td>Height</td>
<td>1.2 m</td>
</tr>
<tr>
<td>W4</td>
<td>Tread</td>
<td>660 mm</td>
</tr>
<tr>
<td>A2</td>
<td>Angle of departure</td>
<td>35°</td>
</tr>
<tr>
<td>L4</td>
<td>Wheelbase</td>
<td>810 mm</td>
</tr>
<tr>
<td>E1</td>
<td>Centreline trench to outside edge of machine, left</td>
<td>381 mm</td>
</tr>
<tr>
<td>E2</td>
<td>Centreline trench to outside edge of machine, right</td>
<td>457 mm</td>
</tr>
<tr>
<td>N</td>
<td>Spoil discharge reach</td>
<td>270 mm</td>
</tr>
<tr>
<td>A3</td>
<td>Angle of approach</td>
<td>85°</td>
</tr>
</tbody>
</table>
4. LOADING CONSIDERATIONS

After demining the test specimen’s geometry (i.e., ECRP with width to length ratio of 2) and size ($\geq 5$ m), the loading method, range, and loading rate should be further evaluated.

4.1 Displacement controlled loading mechanism

Flatjacks have been frequently utilised in previous in-situ tests (Adamson et al., 1995; Cole and Dempsey, 2004; Pennington and Dempsey, 2011; Shapiro and Earl, 1975). In the fracture properties tests, these authors (Adamson et al., 1995; Cole and Dempsey, 2004; Dempsey et al., 1999a; Dempsey et al., 1999h) utilised two loading mechanisms. i.e., 1) they monotonically increased the extension of a nitrogen driven flatjack with load controlled mechanism leading most likely to unstable fractures; or 2) a Crack Mouth Opening Displacement (CMOD) controlled servo-hydraulic system was employed to achieve stable crack propagation (Dempsey et al., 1999a).

In the current test design, we suggest using a screw jack system to achieve a displacement-controlled test. Such a displacement-controlled loading system is expected to ensure a stable crack growth during the test as are theoretically proven in the following.

Based on the weight function method, the CMOD $u(a,0)$ of an ECRP can be written as in Eq. (7) (Dempsey et al., 1995)

$$u(a,0) = \frac{P}{tLE'} \int_0^a h_s^2(s,0) ds$$

in which,
$u(a,0)$ is the normalised CMOD with $u(a,0) = U(A,0)/L$, [-];
$E'$ for a plane stress condition, and $E' = E/(1-\nu^2)$ for a plane strain condition (where $E$ is Young’s modulus and $\nu$ is the Poisson ratio) [kPa];
$h_s(s,0)$ is the normalised weight function of an ECRP with a concentrated force at the crack mouth, [-];

Thus the compliance $C$ of the ECRP can be written as in Eq. (8) and the energy release rate $G$ is followed in Eq. (10) by considering the expression of $h_s(s,0)$ in Eq. (9) (a similar derivations with more details were made by Lu et al. (2015b)).

$$C = \frac{u(a,0)}{P} = \frac{1}{tLE'} \int_0^a h_s^2(s,0) ds$$

The expression of $h_s(s,0)$ can be derived as in Eq. (9) based on the same notations given by Dempsey and Mu (2014).

$$h_s(s,0) = \frac{F_p(s)}{\sqrt{\pi s(1-s)^{3/2}}}$$

$$G(a) = \frac{P^2}{t} \frac{dC}{da} = \frac{P^2}{tLE'} \frac{h_s^2(a,0)}{t^2LE'} = \frac{P^2}{t^2LE'} \frac{F_p^2(a)}{\pi a(1-a)^3}$$

For displacement-controlled test, we replace $P$ in Eq. (10) with its displacement counterpart in Eq. (7), the derivative of the energy release rate $dG(a)/da$ can be rewritten in Eq. (11).
\[
\frac{dG(a)}{da} = u^2(a,0)LE' \frac{2h_r(a,0) \frac{dh_r(a,0)}{da} - 2h_r^4(a,0) \int_0^a h_r^2(s,0) ds}{(\int_0^a h_r^2(s,0) ds)^2}
\] (11)

In order to determine the sign of \(\frac{dG(a)}{da}\), we simply need to determine the sign of a part of the numerator of Eq. (11), i.e., written in a function form of \(g(a)\) in Eq. (12).

\[
g(a) = 2h_r(a,0) \frac{dh_r(a,0)}{da} - 2h_r^4(a,0) \int_0^a h_r^2(s,0) ds
\]

After inserting Eq. (9) into Eq. (12) with some derivative and integral (via Legendre-Gauss Quadrature) manipulations, the results of \(g(a)\) are presented in Figure 5. The figure demonstrates that for the chosen test configuration, a decreasing energy release rate with crack extension (i.e., \(\frac{dG(a)}{da} < 0\)) is expected under such a displacement controlled loading mechanism. Accordingly, crack propagation is expected to take place in a stable manner, e.g., in a form of crack jumps and arrests as described by Dempsey et al. (1999a).

![Figure 5. Values of \(g(a)\) with varying crack length for the chosen test configuration.](image)

![Figure 6. The conceptual design of the screw jack system during operation.](image)
A conceptual design of the screw jack loading system during operation is plotted in Figure 6. The engine prescribes a displacement rate, i.e., 0.3 mm/s (to be introduced in later sections) and hence an expanding force pair is transmitted through this ‘X-shape’ arms down to the ice plate with a 2:1 ratio in displacement and 1:2 ratio in force transmission. In addition, a tension load cell is employed to avoid any slight distortion during the X-shape arms’ expansion. Its measuring range together with the required engine power are discussed in the next section.

4.2 Loading range
Considering the configurations of both the ice specimen and the installed screw jack system, a detailed test system set-up is plotted in Figure 7.

![Figure 7. Detailed configuration of the test specimen.](image)

With this given test configuration, it is necessary to formulate the required loading range and loading rate. The mathematical model utilised for this purpose is illustrated in Figure 8.

![Figure 8. Mathematical models for calculating the loading range and loading rate; a) full scale; b) after normalisation and load profile idealisation.](image)

Figures 7 and 8a both illustrate that an additional wider notch with width \( D \) is needed to install the screw jack. For preliminary estimation purpose, we keep using the weight function of ECRP for the forthcoming calculations. In addition, the distributed pressure \( \Sigma(X) \) is idealised as a concentrated force acting at the centre of the loading area, i.e., at the location of \( X = X_p = D / 2 \). Furthermore, the normalisation procedures in Eq. (13) were implemented in Figure 8b.

\[
\beta = \frac{W}{L}, \quad a = \frac{A}{L}, \quad x_d = \frac{X_d}{L}, \quad x_p = \frac{X_p}{L} \quad (13)
\]

Based on the concept of weight function, the critical force \( P \) acting at \( x = x_p \) can be expressed in Eq. (14) in a non-dimensional format.
The corresponding weight function \( h_{i}(a,x_{p}) \) was derived by Dempsey and Mu (2014) and can also be found in Tsai and Ma (1989). However, it was noted by Dempsey and Mu (2014) that the solution by Tsai and Ma (1989) lose accuracy while the crack length is beyond 60% of the sample size. For a given crack length \( a \), the critical force \( P \) is linearly dependent on the ice thickness, fracture toughness and the square root of specimen length, see in Table 2. Preliminary measurement in February 2015 showed that the ice thickness at the test site is about 56 cm. Experience suggests that the ice thickness will be around 70 cm at the time of the tests and therefore 70 cm thickness is chosen for designing the power of screw jack and choosing an appropriate load cell.

\[
\frac{P}{tK_{Q}\sqrt{L}} = \frac{1}{h_{i}(a,x_{p})}
\]

(14)

Table 2. Estimation of critical force \( P \) based on different ice thickness, specimen size and fracture toughness \( \text{with } a = 0.3 \).

<table>
<thead>
<tr>
<th>( t ) [cm]</th>
<th>( L ) [cm]</th>
<th>Critical force ( P ) [ton]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>700</td>
<td>12.18</td>
</tr>
<tr>
<td>90</td>
<td>700</td>
<td>10.96</td>
</tr>
<tr>
<td>80</td>
<td>700</td>
<td>9.74</td>
</tr>
<tr>
<td>70</td>
<td>700</td>
<td>8.52</td>
</tr>
<tr>
<td>60</td>
<td>700</td>
<td>7.31</td>
</tr>
<tr>
<td>50</td>
<td>700</td>
<td>6.09</td>
</tr>
<tr>
<td>100</td>
<td>600</td>
<td>11.27</td>
</tr>
<tr>
<td>90</td>
<td>600</td>
<td>10.15</td>
</tr>
<tr>
<td>80</td>
<td>600</td>
<td>9.02</td>
</tr>
<tr>
<td>70</td>
<td>600</td>
<td>7.89</td>
</tr>
<tr>
<td>60</td>
<td>600</td>
<td>6.76</td>
</tr>
<tr>
<td>50</td>
<td>600</td>
<td>5.64</td>
</tr>
<tr>
<td>100</td>
<td>500</td>
<td>10.29</td>
</tr>
<tr>
<td>90</td>
<td>500</td>
<td>9.26</td>
</tr>
<tr>
<td>80</td>
<td>500</td>
<td>8.23</td>
</tr>
<tr>
<td>70</td>
<td>500</td>
<td>7.20</td>
</tr>
<tr>
<td>60</td>
<td>500</td>
<td>6.17</td>
</tr>
<tr>
<td>50</td>
<td>500</td>
<td>5.15</td>
</tr>
</tbody>
</table>

A range of ice thickness values within 50~100 cm were used in the calculations, based on which, a tension load cell (i.e., T2075TC2, see Figure 9) with a maximum loading range of 7.5 ton\(^1\) is chosen for this test. In addition, this load cell is built with a signal amplifier device LAU63.1 capable of outputting data higher than 100 Hz.

\(^1\) Recall that there is a 1:2 ratio between the engine’s applying force and the actual critical force acting at the crack mouth (see Figure 6).
4.3 Loading Rate

There are two important ingredients introduced in this test. One is the size of a large enough test specimen; the other is a fast enough loading rate. The loading rate has been emphasised by Schulson and Duval (2009) as one critical factor that has been often neglected in ice’s fracture toughness test.

The loading rate during sea ice’s fracture toughness test is a tricky question. To the author’s knowledge, there is not yet a model proposed in the ice community to describe the development of the so-called creep zone at the crack tip. Only two limiting cases are theoretically studied, i.e., either the creep zone is confined in a small region that LEFM applies; or the primary creep has matured wholly in the bulk material. Nevertheless, we consider the following three conflicting factors in relation to a critical loading rate:

- The test sample should be loaded fast enough that the creep zone is confined in a small region;
- We should not load the test sample too fast to introduce dynamic effect, i.e., stress wave interference with the stress state at the crack tip;
- Theoretical analysis shows that a larger specimen needs higher loading rate to ensure the applicability of LEFM (Mulmule and Dempsey, 2000). It is therefore necessary to make a balanced choice between the loading rate and test specimen size.

We treat each of these factors in details as shown below.

Table 3. Calculated and estimated critical loading rate for the fracture toughness test of freshwater and sea ice respectively under different temperatures.

(from Nixon and Schulson (1987))

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>Calculated $K_I^*$ for freshwater ice kPa√m/s</th>
<th>Estimated $K_I^*$ for sea ice kPa√m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50</td>
<td>0.005</td>
<td>0.05</td>
</tr>
<tr>
<td>-20</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>-10</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>-5</td>
<td>100</td>
<td>1000</td>
</tr>
<tr>
<td>-2</td>
<td>200</td>
<td>2000</td>
</tr>
</tbody>
</table>

Following the model of Riedel and Rice (1980), Nixon and Schulson (1988) worked out the loading rate requirement to impede creep effect in fresh water ice, i.e., the applicability of LEFM. In addition, it was mentioned that for sea ice, the loading rate is expected to be a level
of magnitude higher. These calculated critical loading rate values (not including the experimental observed value ranges) for fresh water ice and estimated sea ice are presented in Table 3. Experience shows that the temperature on the test date may be very well below -10 °C. Therefore we are targeting a loading rate which is larger than 20 kPa√m/s.

In terms of the dynamic effect induced by a rapid loading, Nakamura et al. (1986) proposed a transient time \( t_\tau \) concept. Many dynamically loaded specimen can be characterised by a short-time response (test duration \( T < t_\tau \) ) dominated by discrete waves and a long-time response that can be characterised as quasistatic (Anderson, 2005). This transition time is estimated as in Eq. (15)

\[
t_\tau \approx 27(L-A) / c_1
\]

in which

\( c_1 \)

is the longitudinal wave speed in an unbounded solid [m/s].

If we approximate the wave speed \( c_1 = \sqrt{E/\rho_{\text{ice}}} \) with \( \rho_{\text{ice}} = 920 \, \text{kg/m}^3 \) which is the density of sea ice, the transition time for the test specimen configuration in Figure 7 can be easily calculated. In addition, it was further suggested in Anderson (2005) that the test duration \( T \) should be larger than 2\( t_\tau \) to ensure a quasistatic test. Based on this reasoning process, our maximum loading rate is expressed in Table 4.

\[
\dot{K}_{l,max}^* < \frac{K^*_Q}{2t_\tau}
\]

Table 4. Maximum loading rate based on the transition time concept to avoid inertia effect during rapid loading.

<table>
<thead>
<tr>
<th>( E ) [GPa]</th>
<th>( c_1 ) [m/s]</th>
<th>( t_\tau ) [s]</th>
<th>( 2t_\tau ) [s]</th>
<th>( \dot{K}_{l,max}^* ) [kPa√m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1805.788</td>
<td>0.073</td>
<td>0.147</td>
<td>1706</td>
</tr>
<tr>
<td>4</td>
<td>2085.144</td>
<td>0.063</td>
<td>0.127</td>
<td>1970</td>
</tr>
<tr>
<td>5</td>
<td>2331.262</td>
<td>0.057</td>
<td>0.114</td>
<td>2202</td>
</tr>
</tbody>
</table>

Comparing the maximum allowed loading rate in Table 4 against the minimum loading rate in Table 3, one immediately see that this test can only be carried out below -10°C to have a better chance in fulfilling these two constraints simultaneously.

Next, we need to relate the loading rate \( \dot{K}_{l,max}^* \) with the displacement rate at \( x = x_p \) (see Figure 8) such that the engine can drive the screw jack with the desired rate of loading.

Based on the concept of weight function method, the displacement \( u(a, \xi_d) \) at \( x = \xi_d \) \((0 \leq \xi < a)\) under a pair of concentrated force \( P \) acting at \( x = \xi_p \) can be written as in Eq. (17).

\[
u(a, \xi_d) = \frac{P}{tLE} \int_{\max(\xi_d, \xi_p)}^{a} h_1(s, \xi_d)h_2(s, \xi_p)ds
\]

Inserting Eq. (14) into the above equation, a unique relationship between displacement \( U(A, X_d) \) and stress intensity factor \( K_Q(A) \) can be established as in Eq. (18).
If we replace $K_0(A)$ in Eq. (18) with $K_0^*$, the corresponding maximum displacement $2U(A,\xi_d)$ can be calculated. If we write Eq. (18) in incremental form (with time), the loading rate relationship is thus established in Eq. (19).

$$
U(A, X_d) = \frac{K_0(A)\sqrt{L}}{E'} \frac{1}{h_r(a, X_p)} \int_{\max(\xi_d, \xi_p)}^{\xi} h_r(s, \xi_d)h_r(s, \xi_p)ds
$$

If we replace $K_0(A)$ in Eq. (18) with $K_0^*$, the corresponding maximum displacement $2U(A,\xi_d)$ can be calculated. If we write Eq. (18) in incremental form (with time), the loading rate relationship is thus established in Eq. (19).

$$
\dot{U}(A, X_d) = \frac{\dot{K}_0(A)\sqrt{L}}{E'} \frac{1}{h_r(a, X_p)} \int_{\max(\xi_d, \xi_p)}^{\xi} h_r(s, \xi_d)h_r(s, \xi_p)ds
$$

Eq. (19) quantitatively demonstrates that to achieve a given loading rate $\dot{K}_0(A)$, a faster loading rate $\dot{U}(A, X_d)$ is required for a larger specimen size. Following the test configuration in Figure 7 and assuming $A=0.3L=2.1\text{m}$, the following calculations regarding maximum Crack Opening Displacement (COD) and recommended loading rates are shown in Tables 5 and 6 respectively.

**Table 5. Maximum COD at the loading location $X = X_p$ and measuring location $X = X_d$ (see Figure 8).**

<table>
<thead>
<tr>
<th>$E$ [GPa]</th>
<th>$2U(A, X_p)$ [mm]</th>
<th>$2U(A, X_d)$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_0^* = K_{0,F}$</td>
<td>$K_0^* = K_{0,L}$</td>
</tr>
<tr>
<td>3</td>
<td>1.7201</td>
<td>0.7912</td>
</tr>
<tr>
<td>4</td>
<td>1.2901</td>
<td>0.5934</td>
</tr>
<tr>
<td>5</td>
<td>1.032</td>
<td>0.4747</td>
</tr>
</tbody>
</table>

**Table 6. Required loading rate**

<table>
<thead>
<tr>
<th>$E$ [GPa]</th>
<th>$\dot{2U}(A, X_d)$ [mm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\dot{K}_0 = 20 \text{kPa}\sqrt{\text{m/s}}$</td>
</tr>
<tr>
<td>3</td>
<td>0.1376</td>
</tr>
<tr>
<td>4</td>
<td>0.1032</td>
</tr>
<tr>
<td>5</td>
<td>0.0826</td>
</tr>
</tbody>
</table>

Judging from the maximum COD at $X = X_d$ in Table 5, it is suggested that a displacement measuring device with a measuring range of 0~1 mm and with a resolution at least 0.01 mm is required.

Meanwhile, Table 6 demonstrates that Young’s modulus greatly influences the required loading rate. Bearing in mind our target loading rate, i.e., $\dot{K}_0 = 20 \text{kPa}\sqrt{\text{m/s}}$ and also a potentially encountered Young’s modulus between 3-4 GPa, we decide to choose a loading rate of $2\dot{U}(A, X_d) = 0.15 \text{mm/s}$. After the transmission of the X-shape arms, this means that the screw jack should output a displacement rate of 0.3 mm/s (see in Figure 6).
5. CONCLUSIONS
This paper documented a detailed theoretical analysis while designing a test campaign in extracting the fracture toughness of sea ice. A simplified test methodology based on Linear Elastic Fracture Mechanics (LEFM) was adopted in the test design. The key issue to be resolved in this paper is that we need test on a large enough specimen with fast enough loading rate. Thus, the quantification process of specimen size and loading rate covers the major portion of this paper. During the design process, we offered the following specifications for the first test campaign:

- Based on notch sensitivity analysis, a specimen with length of 7 m and a width to length ratio of 2 is recommended in the test;
- A displacement controlled screw jack system is suggested to ensure stable crack propagation. This is beneficial for the future more in-depth test campaigns;
- A loading rate of 0.15 mm/s (i.e., 0.3 mm/s for the screw jack) at the loading area is adopted to potentially impede the creep zone’s size increase based on relevant recommendations;
- The test is preferably executed at temperatures lower than -10°C. Otherwise, quite demanding loading rate and data logger system is required;
- The displacement range of two potential points near the crack mouth was calculated. The values suggest that a load sensor with a measuring range of 0-1 mm and a minimum resolution of 0.01 mm is required.

Apart from the above practical recommendations, the theoretical calculations are expected to potentially serve as an example for future design processes. Interesting results were also obtained during the theoretical analysis. These include:

- Based on a transition time concept in dynamic fracture mechanics, the maximum loading rates were calculated in Table 4. The conflict between impeding creep with fast loading and avoiding dynamic effect with slow loading was highlighted, particularly at warmer temperatures;
- In addition, the derivation of Eq. (19) sheds light on the relationship between the displacement controlled loading rate and the rate of stress intensity factor;
- Eq. (19) quantified the statement: ‘faster loading rate is required for a larger specimen’.

The test itself is a continuing trial and error process. We have greatly benefitted from precursors’ test experiences and pertinent theoretical development. The test itself has not yet been carried out by the time of this paper’s completion. The upcoming test results and experience shall be reported in a separate paper.

ACKNOWLEDGEMENT
The authors would like to thank the Norwegian Research Council through the research centre SAMCoT CRI for financial support and all the SAMCoT partners. In addition, valuable discussions and suggestions from Professor John Dempsey is greatly acknowledged.

REFERENCES


